LOSS COVERAGE IN INSURANCE MARKETS: WHY ADVERSE SELECTION IS NOT ALWAYS A BAD THING

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Abstract

This paper investigates equilibrium in an insurance market where risk classification is restricted. Insurance demand is characterised by an iso-elastic function with a single elasticity parameter. We characterise the equilibrium by three quantities: equilibrium premium; level of adverse selection; and "loss coverage", defined as the expected population losses compensated by insurance. We find that equilibrium premium and adverse selection increase monotonically with demand elasticity, but loss coverage first increases and then decreases. We argue that loss coverage represents the efficacy of insurance for the whole population; and therefore, if demand elasticity is sufficiently low, adverse selection is not always a bad thing.

1. Introduction

Restrictions on insurance risk classification can lead to troublesome adverse selection. A simple version of the usual argument is as follows. If insurers cannot charge risk-differentiated premiums, more insurance is bought by higher risks and less insurance is bought by lower risks. This raises the equilibrium pooled price of insurance above a population-weighted average of true risk premiums. Also, since the number of higher risks is usually smaller than the number of lower risks, the total number of risks insured usually falls. This combination of a rise in price and fall in demand is usually portrayed as a bad outcome, both for insurers and for society.

However, it can be argued that from a social perspective, higher risks are those more in need of insurance. Also, the compensation of many types of loss by insurance appears to be widely regarded as a desirable objective, which public policymakers often seek to promote (for example by tax relief on premiums). Insurance of one higher risk contributes more in expectation to this objective than insurance of one lower risk. This suggests that public policymakers might welcome increased purchasing by higher risks, except for the usual story about adverse selection.

The usual story about adverse selection overlooks one point: with adverse selection, expected losses compensated by insurance can be *higher* than with no adverse selection. The rise in equilibrium price with adverse selection reflects a shift in coverage towards higher risks. From a public policymaker's viewpoint, this means that more of the "right" risks, i.e. those more likely to suffer loss, buy insurance. If the shift in coverage is large enough, it can more than outweigh the fall in numbers insured. This result of higher expected losses compensated by insurance, i.e. higher "loss coverage", might be regarded by a public policymaker as a better outcome for society than that obtained with no adverse selection.

Another way of putting this is that a public policymaker designing risk classification policies in the context of adverse selection normally faces a trade-off between insurance of the "right" risks (those more likely to suffer loss), and insurance of a larger number of risks. The optimal trade-off depends on demand elasticities in higher and lower risk-groups, and will normally involve at least some adverse selection. The concept of loss coverage quantifies this trade-off, and provides a metric for comparing the effects of different risk classification schemes.

2. Motivating Examples

We now give three heuristic examples of insurance market equilibria to illustrate the concept of loss coverage and the possibility that loss coverage may be increased by some adverse selection.

Suppose that in a population of 1,000 risks, 16 losses are expected every year. There are two risk-groups. The high risk-group of 200 individuals has a probability of loss 4 times higher than those in the low risk-group. This is summarised in Table 1.

	Low risk-group	High risk-group	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums (differentiated)	0.01	0.04	0.016
Numbers insured	400	100	500
Insured losses	4	4	8
Loss coverage			0.5

Table 1: Full risk classification with no adverse selection.

We further assume that probability of loss is not altered by the purchase of insurance, i.e. there is no moral hazard. An individual's risk-group is fully observable to insurers and all insurers are required to use the same risk classification regime. The equilibrium, or "break-even", price of insurance is determined as the price at which insurers make zero profit.

Under our first risk classification regime, insurers operate full risk classification, charging actuarially fair premiums to members of each risk-group. We assume that the proportion of each risk-group which buys insurance under these conditions, i.e. the "fair-premium proportional demand", is 50%. Table 1 shows the outcome, which can be summarised as follows:

(a) There is no adverse selection, as premiums are actuarially fair and the demand is at the fair-premium proportional demand.

(b) Half the losses in the population are compensated by insurance. We heuristically characterise this as a "loss coverage" of 0.5.

Now suppose that a new risk classification regime is introduced, where insurers have to charge a single "pooled" price to members of both the low and high risk-groups. One possible outcome is shown in Table 2, which can be summarised as follows:

(a) The pooled premium of 0.02 at which insurers make zero profits is calculated as the demand-weighted average of the risk premiums: $(300 \times 0.01 + 150 \times 0.04)/450 = 0.02$.

(b) The pooled premium is expensive for low risks, so fewer of them buy insurance (300, compared with 400 before). The pooled premium is cheap for high risks, so more of them buy insurance (150, compared with 100 before). Because there are 4 times as many low risks as high risks in the population, the total number of policies sold falls (450, compared with 500 before).

(c) There is moderate adverse selection, as the break-even pooled premium exceeds population-weighted average risk and the aggregate demand has fallen.

(d) The resulting loss coverage is 0.5625. The shift in coverage towards high risks more than outweighs the fall in number of policies sold: 9 of the 16 losses (56%) in the population as a whole are now compensated by insurance (compared with 8 of 16 before).

	Low risk-group	High risk-group	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums (pooled)	0.02	0.02	0.02
Numbers insured	300	150	450
Insured losses	3	6	9
Loss coverage			0.5625

Table 2: No risk classification leading to moderate adverse selection but higher loss coverage.

Another possible outcome under the restricted risk classification scheme, this time with more severe adverse selection, is shown in Table 3, which can be summarised as follows:

(a) The pooled premium of 0.02154 at which insurers make zero profits is calculated as the demand-weighted average of the risk premiums: $(200 \times 0.01 + 125 \times 0.04)/325 = 0.02154$.

(b) There is severe adverse selection, with further increase in pooled premium and significant fall in demand.

(c) The loss coverage is 0.4375. The shift in coverage towards high risks is insufficient to outweigh the fall in number of policies sold: 7 of the 16 losses (43.75%) in the population as a whole are now compensated by insurance (compared with 8 of 16 in Table 1, and 9 out of 16 in Table 3).

	Low risk-group	High risk-group	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums (pooled)	0.02154	0.02154	0.02154
Numbers insured	200	125	325
Insured losses	2	5	7
Loss coverage			0.4375

Table 3: No risk classification leading to severe adverse selection and lower loss coverage.

Taking the three tables together, we can summarise by saying that compared with an initial position of no adverse selection (Table 1), moderate adverse selection leads to a higher fraction of the population's losses compensated by insurance (higher loss coverage) in Table 2; but too much adverse selection leads to a lower fraction of the population's losses compensated by insurance (lower loss coverage) in Table 3.

This argument is quite general: it does not depend on any unusual choice of numbers for the examples. It also does not assume any preference by the policymaker for compensating losses of the low or high risk-groups. The same preference is given to compensation of losses anywhere in the population *ex-post*, when all uncertainty about who will suffer a loss has been resolved. This implies giving higher preference to insurance cover for higher risks *ex-ante*, but only in proportion to their higher risk.

3. Model

Based on the motivation in the previous section, we now develop a model to analyse the impact of restricted risk classification on equilibrium premium, adverse selection and loss coverage. We first outline the model assumptions and define the underlying concepts.

3.1. Population Parameters

We assume that a population of risks can be divided into a high risk-group and a low risk-group, based on information which is fully observable by insurers. Let μ_1 and μ_2 be the underlying risks (probabilities of loss). Let p_1 and p_2 be the respective population fractions, i.e. a risk chosen at random from the entire population has a probability of p_i of belonging to the risk-group $i = 1, 2$.

For simplicity, we assume that all losses are of unit size. All quantities defined below are for a single risk sampled at random from the population (unless the context requires otherwise).

The expected loss is given by:

$$E[L] = \mu_1 p_1 + \mu_2 p_2, \tag{1}$$

where L is the loss for a risk chosen at random from the entire population.

Information on risk being freely available, insurers can distinguish between the two risk-groups accurately and charge premiums π_1 and π_2 for risks in risk-groups 1 and 2 respectively.

The expected insurance coverage is given by:

$$E[Q] = d(\mu_1, \pi_1)p_1 + d(\mu_2, \pi_2)p_2, \tag{2}$$

where $d(\mu_i, \pi_i)$ denotes the proportional demand for insurance for risk-group i at premium π_i, i.e. the probability that an individual selected at random from risk-group i buys insurance.

The expected premium is given by:

$$E[Q\pi] = d(\mu_1, \pi_1)p_1\pi_1 + d(\mu_2, \pi_2)p_2\pi_2, \tag{3}$$

where π is π_1 or π_2 with probability p_1 or p_2 respectively.

The expected insurance claim, i.e. the loss coverage, is given by:

$$\text{Loss coverage: } E[QL] = d(\mu_1, \pi_1)p_1\mu_1 + d(\mu_2, \pi_2)p_2\mu_2, \tag{4}$$

where we assume no moral hazard, i.e. purchase of insurance has no bearing on the risk. Loss coverage can also be thought of as *risk-weighted* insurance demand.

Finally, dividing Equation 4 by Equation 2 we obtain the expected claim per policy, say $\rho(\pi_1, \pi_2)$, which is given by:

$$\text{Expected claim per policy: } \rho(\pi_1, \pi_2) = \frac{E[QL]}{E[Q]} \tag{5}$$

3.2. Demand for insurance

In the previous section, we have introduced the concept of proportional demand for insurance, $d(\mu_i, \pi_i)$, when a premium π_i is charged for risk-group with true risk μ_i. In this section, we specify a functional form for $d(\mu_i, \pi_i)$ and its relevant properties.

De Jong and Ferris (2006) suggested axioms for an insurance demand function, adapted below using our notations:

(a) $d(\mu_i, \pi_i)$ is a decreasing function of premium π_i for all risk-groups i;

(b) $d(\mu_1, \pi) < d(\mu_2, \pi)$, i.e. at a given premium π, the proportional demand is greater for the higher risk-group;

(c) $d(\mu_i, \pi_i)$ is a decreasing function of the premium loading π_i/μ_i; and

(d) for our model, where all losses are of unit size, we need to add $d(\mu_i, \pi_i) < 1$, i.e. the highest possible demand is when all members of the risk-group buy insurance.

These authors suggested a "flexible but practical" exponential-power demand function, and this approach was also followed by Thomas (2008, 2009). However the exponential-power function, whilst very flexible, is also rather intractable. In the present paper, we use a more tractable function which satisfies the axioms above and for which the price elasticity of demand λ, is a constant for all risk-groups, i.e.:

$$-\frac{\pi_i}{d(\mu_i, \pi_i)} \frac{\partial d(\mu_i, \pi_i)}{\partial \pi_i} = \lambda. \tag{6}$$

Solving Equation 6 leads to the "iso-elastic" demand function:

$$d(\mu_i, \pi_i) = \tau_i \left(\frac{\pi_i}{\mu_i} \right)^{-\lambda}, \tag{7}$$

where $\tau_i = d(\mu_i, \mu_i)$ is the "fair-premium demand" for insurance for risk-group i, that is the probability that a member sampled randomly from risk-group i buys insurance when premiums are actuarially fair.

The formula specifies demand as a function of the "premium loading" (π/μ_i). When the premium loading is high (insurance is expensive), demand is low, and vice versa. The λ parameter controls the shape of the demand curve. The "iso-elastic" terminology reflects that the price elasticity of demand is the same everywhere along the demand curve.

Clearly, iso-elastic demand functions satisfy axioms (a) and (c) of De Jong and Ferris (2006). Axioms (b) and (d) appear superficially to require conditions on the fair-premium demands τ_1 and τ_2. However, if we define *fair-premium demand-shares* α_1 and α_2 as:

$$\text{Fair-premium demand-share: } \alpha_i = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}, \quad i = 1, 2 \tag{8}$$

then it turns out that that α_1 and α_2 fully specify the population structure in the form required for our model. Since increasing the τ_i is mathematically equivalent to decreasing the p_i and vice versa, we do not need to specify any particular values for them. We can analyse the model for the full range of fair-premium demand-shares $0 < \alpha_i < 1$, knowing that for every α_i there must exist some corresponding combination of p_i and τ_i which satisfies the axioms (b) and (d) above.

4. Equilibrium

In the model in Section 3, an insurance market equilibrium is reached when the premiums charged (π_1, π_2) ensure that the expected profit, $f(\pi_1, \pi_2) = 0$, where:

$$\begin{aligned} f(\pi_1, \pi_2) &= E[Q\pi] - E[QL] && (9) \\ &= d(\mu_1, \pi_1)(\pi_1 - \mu_1)p_1 + d(\mu_2, \pi_2)(\pi_2 - \mu_2)p_2. && (10) \end{aligned}$$

4.1. Risk-differentiated Premiums

An obvious solution to the profit equation $f(\pi_1, \pi_2) = 0$ is to set $(\pi_1, \pi_2) = (\mu_1, \mu_2)$, i.e. setting premiums equal to the respective risks results in an expected profit of zero for each risk group and also in aggregate. We shall refer to this case as *risk-differentiated premiums*.

Following the notations introduced in Section 3, the expected insurance coverage is given by:

$$E[Q] = \tau_1 p_1 + \tau_2 p_2. \tag{11}$$

Also, $(\pi_1, \pi_2) = (\mu_1, \mu_2)$, i.e. the expected premium and expected claim are equal and given by:

$$E[Q\pi] = E[QL] = \tau_1 p_1 \mu_1 + \tau_2 p_2 \mu_2. \tag{12}$$

So, the expected claim per policy is:

$$\rho(\mu_1, \mu_2) = \frac{E[QL]}{E[Q]} = \frac{\tau_1 p_1 \mu_1 + \tau_2 p_2 \mu_2}{\tau_1 p_1 + \tau_2 p_2} = \alpha_1 \mu_1 + \alpha_2 \mu_2. \tag{13}$$

4.2. Pooled Premium

Next we consider the case of *pooled premium*. This is where risk classification is banned, so that insurers have to charge the same premium π_0 for both risk-groups, i.e. $\pi_1 = \pi_2 = \pi_0$, leading to $f(\pi_0, \pi_0) = 0$. For convenience, we omit one argument for all bivariate functions if both arguments are equal, e.g. we write $f(\pi)$ for $f(\pi, \pi)$.

Equation 9 leads to the following relationship for the equilibrium premium π_0:

$$\pi_0 = \frac{E[QL]}{E[Q]}. \tag{14}$$

The existence of a solution for $f(\pi) = 0$ within the interval $(\mu_1, \mu_2]$ is obvious, because $f(\pi)$ is a continuous function with $f(\mu_1) < 0$ and $f(\mu_2) \geq 0$. Assuming an iso-elastic demand function with constant elasticity of demand, λ, Equation 14 provides a unique solution:

$$\pi_0 = \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1^{\lambda} + \alpha_2 \mu_2^{\lambda}}. \tag{15}$$

This can be written as a weighted average of the true risks μ_1 and μ_2:

$$\pi_0 = v\mu_1 + (1 - v)\mu_2, \quad \text{where} \quad v = \frac{\alpha_1}{\alpha_1 + \alpha_2 \left(\frac{\mu_2}{\mu_1}\right)^{\lambda}}. \tag{16}$$

Note that π_0 does not depend directly on the individual values of the population fractions (p_1, p_2) and fair-premium demands (τ_1, τ_2), but only indirectly on these parameters through the demand-shares (α_1, α_2). So, populations with the same true risks (μ_1, μ_2) and demand-shares (α_1, α_2) have the same equilibrium premium, even if the underlying (p_1, p_2) and (τ_1, τ_2) are different.

Figure 1 shows the plots of pooled equilibrium premium against demand elasticity, λ, for two different population structures with the same true risks $(\mu_1, \mu_2) = (0.01, 0.04)$ but different fair-premium demand-shares (α_1, α_2). The following observations can be derived from Equations 15 and 16, and are illustrated by Figure 1:

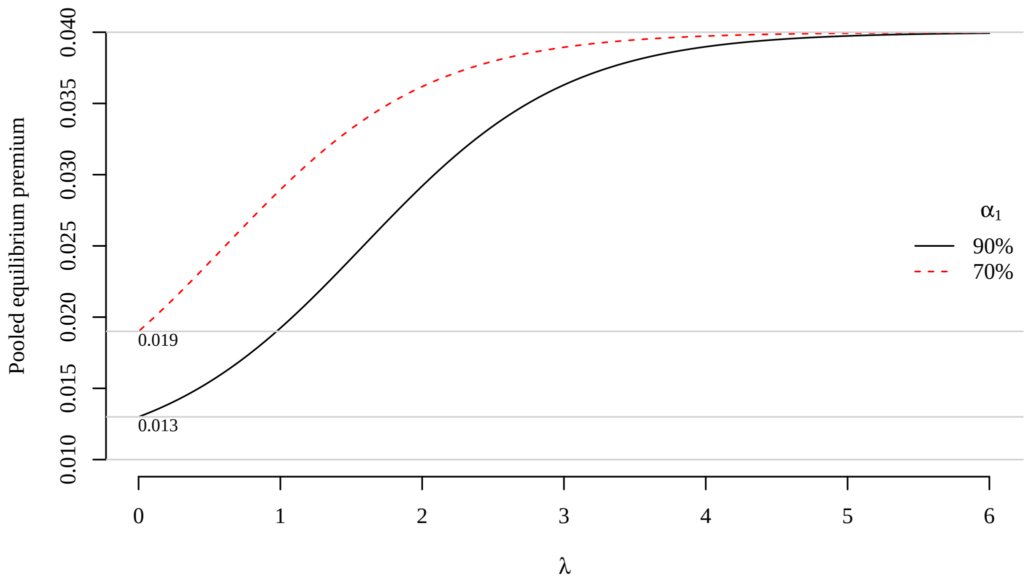

Figure 1: Pooled equilibrium premium as a function of λ for two populations with the same $(\mu_1, \mu_2) = (0.01, 0.04)$ but different values of α_1.

(a) $\lim_{\lambda\to 0} \pi_0 = \alpha_1\mu_1 + \alpha_2\mu_2 = \rho(\mu_1, \mu_2)$. If demand is inelastic, changing premium has no impact on demand, and so the equilibrium premium will be the same as the expected claim per policy if risk-differentiated premiums were charged.

(b) π_0 is an increasing function of λ. In Equation 16, increasing λ reduces the weight w on low-risk, resulting in an increase in the equilibrium premium π_0.

(c) $\lim_{\lambda\to\infty} \pi_0 = \mu_2$. If demand elasticity is very high, demand from the low risk-group falls to zero for any premium above their true risk μ_1. The only remaining insureds are then all high risks, so the equilibrium premium must move to $\pi_0 = \mu_2$.

(d) π_0 is a decreasing function of α_1. If the fair-premium demand-share α_1 of the lower risk-group increases, we would expect the equilibrium premium to fall.

5. Adverse Selection

Adverse selection is typically defined in the economics literature as a positive correlation (or equivalently, covariance) of coverage and losses (e.g. for a survey see Cohen and Siegelman (2010)). Using the notations developed in Section 3, this can be quantified by the covariance between the random variables Q and L, i.e. $E[QL] - E[Q]E[L]$. We prefer to use the ratio rather than the difference, so our definition is:

$$\text{Adverse selection} = \frac{E[QL]}{E[Q]E[L]} = \frac{\rho(\pi_1, \pi_2)}{E[L]}, \tag{17}$$

where $\rho(\pi_1, \pi_2)$ is the expected claim per policy as defined in Equation 5. This metric for adverse selection is intuitively appealing: it is the the ratio of the *expected claim per policy* to the *expected loss per risk*, where the risk is drawn at random from the whole population.

To compare the severity of adverse selection under different risk classification regimes, we need to define a reference level of adverse selection. We use the level under risk-differentiated premiums, $\rho(\mu_1, \mu_2)/E[L]$, and so define the *adverse selection ratio* as:

$$\text{Adverse selection ratio: } S(\pi_1, \pi_2) = \frac{\rho(\pi_1, \pi_2)}{\rho(\mu_1, \mu_2)} = \frac{\rho(\pi_1, \pi_2)}{\alpha_1\mu_1 + \alpha_2\mu_2}. \tag{18}$$

Note that as the same underlying population is used in both the numerator and the denominator of the ratio, the population expected loss $E[L]$ gets cancelled and does not play any further role.

An interesting case arises for pooled equilibrium premium, where by Equation 14, an equilibrium premium π_0 satisfies the condition:

$$\pi_0 = \frac{E[QL]}{E[Q]} = \rho(\pi_0, \pi_0). \tag{19}$$

So in the particular case of pooled equilibrium premium, π_0, we have:

$$\text{Adverse selection ratio: } S(\pi_0) = \frac{\pi_0}{\alpha_1\mu_1 + \alpha_2\mu_2}. \tag{20}$$

In essence, the pooled equilibrium premium itself (scaled by the expected claim per policy under risk-differentiated premiums) provides a measure of the adverse selection.

Figure 2 shows the adverse selection ratio under pooling for two populations with the same underlying risks $(\mu_1, \mu_2) = (0.01, 0.04)$ and demand elasticity λ, but different values of the fair-premium demand-share α_1.

The following properties of the adverse selection ratio, $S(\pi_0)$, follow directly from the observations in Section 4:

(a) $S(\pi_0) \geq 1$, as the pooled equilibrium premium, π_0, is always higher than the expected claim per policy for risk-differentiated premiums.

(b) $S(\pi_0)$ is an increasing function of the underlying demand elasticity.

(c) $\lim_{\lambda \to \infty} S(\pi_0) = \frac{\mu_2}{\alpha_1\mu_1 + \alpha_2\mu_2}$, i.e. when demand is very elastic, the adverse selection ratio tends towards a limit where only higher risks are insured.

The adverse selection ratio is always higher under pooling than under risk-differentiated premiums. It also increases monotonically with the underlying demand elasticity. Therefore this metric is unable to distinguish between cases where pooling gives a better outcome for society as a whole (Table 2 in the motivating examples in Section 2) and cases where pooling gives a worse outcome for society as a whole (Table 3 in the motivating examples in Section 2). This leads us to the concept of loss coverage ratio discussed in the next section.

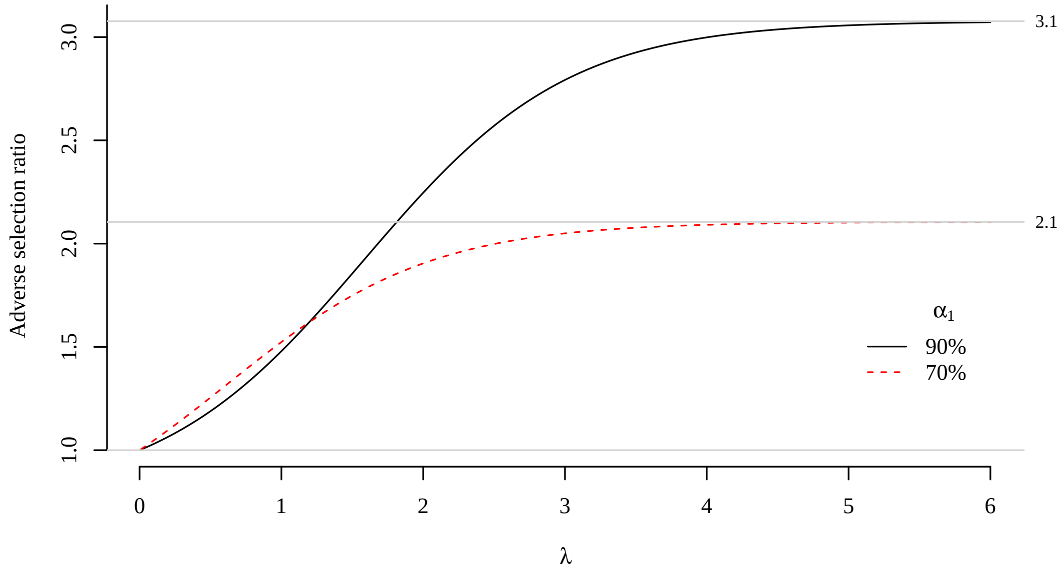

Figure 2: Adverse selection ratio as a function of λ for two populations with the same $(\mu_1, \mu_2) = (0.01, 0.04)$ but different values of α_1.

6. Loss Coverage

The motivating examples in Section 2 suggested loss coverage, heuristically characterised as the proportion of the population's losses compensated by insurance, as a measure of the social efficacy of insurance. This can be formally quantified in our model by the expected insurance claim, $E[QL]$, as defined in Section 3 as:

$$\text{Loss coverage: } LC(\pi_1, \pi_2) = E[QL]. \tag{21}$$

To compare the relative merits of different risk classification regimes, we define a reference level of loss coverage. We use the level under risk-differentiated premiums (i.e. the same approach as for adverse selection in Equation 18), and so define the *loss coverage ratio*, as follows:

$$\text{Loss coverage ratio: } C = \frac{LC(\pi_1, \pi_2)}{LC(\mu_1, \mu_2)}. \tag{22}$$

Considering loss coverage ratio for pooled premium, i.e $\pi_1 = \pi_2 = \pi_0$, and using the iso-elastic demand function with demand elasticity λ, in Equation 4, gives:

$$C(\lambda) = \frac{1}{\pi_0^{\lambda}} \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1 + \alpha_2 \mu_2}, \tag{23}$$

where π_0 is the pooled equilibrium premium given in Equation 15. The above can also be conveniently re-expressed as:

$$C(\lambda) = \frac{\left[w\beta^{1-\lambda} + (1-w)\right]^{\lambda} \left[w + (1-w)\beta^{\lambda}\right]^{1-\lambda}}{\beta^{\lambda(1-\lambda)}}, \text{ where} \tag{24}$$
$$w = \frac{\alpha_1\mu_1}{\alpha_1\mu_1 + \alpha_2\mu_2} \quad \text{and} \quad \beta = \frac{\mu_2}{\mu_1} > 1.$$

Figure 3 shows loss coverage ratio for four population structures. Both plots in Figure 3 show the same example, with the right-hand plot zooming over the range $0 < \lambda < 1$.

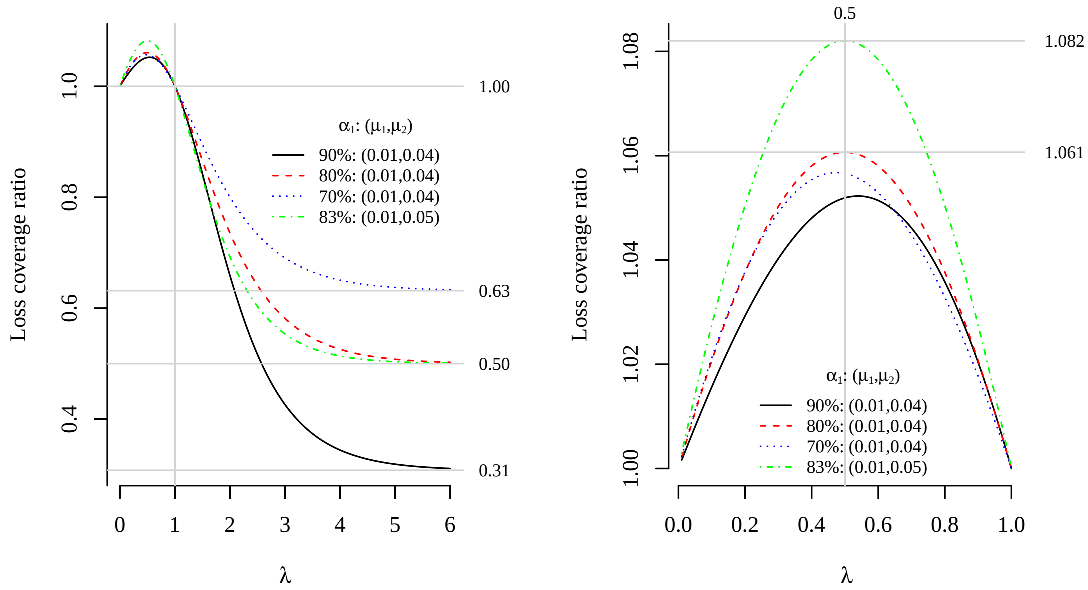

Figure 3: Loss coverage ratio as a function of λ for four population structures.

We make the following observations:

(a) $\lim_{\lambda\to 0} C(\lambda) = 1$. This follows directly from Equation 23. Intuitively, if demand is inelastic then pooling must give the same loss coverage as fair premiums.

(b) $\lim_{\lambda\to\infty} C(\lambda) = 1 - w = \frac{\alpha_2\mu_2}{\alpha_1\mu_1+\alpha_2\mu_2}$. This follows from Equation 24. Recall that for highly elastic demand, equilibrium is achieved when only high risks buy insurance at the equilibrium premium $\pi_0 = \mu_2$, which explains the above result.

(c) For $\lambda > 0$,

$$\lambda \lesseqgtr 1 \Rightarrow C(\lambda) \gtreqless 1. \tag{25}$$

The result implies that pooling produces higher loss coverage than fair premiums if demand elasticity is less than 1. The proof of this result is given in the Appendix.

(d)

$$\max_{w,\lambda} C = \frac{1}{2}\left(\sqrt[4]{\frac{\mu_2}{\mu_1}} + \sqrt[4]{\frac{\mu_1}{\mu_2}}\right) = \frac{1}{2}\left(\sqrt[4]{\beta} + \frac{1}{\sqrt[4]{\beta}}\right). \tag{26}$$

As can be seen from the right-hand plot of Figure 3, for a given value of relative risk, β, loss coverage ratio attains its maximum when $\lambda = 0.5$ and $w = 0.5$. Moreover, the maximum loss coverage ratio increases with increasing relative risk. This implies that a pooled premium might be highly beneficial in the presence of a small group with very high risk exposure. The proof of this result is given in the Appendix.

7. Summary and Discussion

The results in preceding sections can be summarised and interpreted as follows.

Loss coverage is the expected insurance claim, that is the expected losses compensated by insurance. Loss coverage can also be thought of as the *risk-weighted* insurance demand.

Adverse selection is always higher under pooling than under risk-differentiated premiums. On the other hand, loss coverage can be higher or lower under pooling than under risk-differentiated premiums. Loss coverage is higher under pooling if the shift in coverage towards higher risks more than compensates for the fall in number of risks insured.

For iso-elastic demand with equal demand elasticities in high and low risk-groups, equilibrium premium and loss coverage ratio can be characterised as follows:

(a) Equilibrium premium (and hence adverse selection) under pooling increases monotonically with demand elasticity, tending to an upper limit where the only remaining insureds are high risks.

(b) Loss coverage ratio under pooling is always greater than 1 for demand elasticity less than 1.

(c) As demand elasticity increases from zero, loss coverage ratio increases to a maximum for demand elasticity around 0.5; then decreases to 1 when demand elasticity is 1; and then flattens out at a lower limit for high demand elasticity, where the only remaining insureds are high risks.

(d) The maximum value of loss coverage ratio, which is reached when demand elasticity is around 0.5, depends on the relative risk, β (probability of loss for high risks divided by that for low risks). A higher relative risk gives a higher maximum for loss coverage ratio.

Loss coverage represents the expected losses compensated by insurance for the whole population, which we have suggested is a reasonable metric for the social efficacy of insurance. If this suggestion is accepted, and if our iso-elastic model of insurance demand is reasonable, then pooling will be beneficial whenever demand elasticity is less than 1.

There is some empirical evidence that insurance demand elasticities are typically less than 1 in many markets. We defined demand elasticity as a positive constant in Equation 6, but the estimates in empirical papers are generally given with the negative sign, and so we quote them in that form. For example, for yearly renewable term insurance in the US, an estimate of -0.4 to -0.5 has been reported (Pauly et al. (2003)). A questionnaire survey about life insurance purchasing decisions produced an estimate of -0.66 (Viswanathan et al. (2007)). For private health insurance in the

US, several studies estimate demand elasticities in the range of 0 to -0.2 (Chernew et al. (1997); Blumberg et al. (2001); Buchmueller and Ohri (2006)). For private health insurance in Australia, Butler (1999) estimates demand elasticities in the range -0.36 to -0.50. These magnitudes are consistent with the possibility that loss coverage can sometimes be increased by restricting risk classification.

Our model considers only two possibilities for risk classification, fully risk-differentiated premiums or complete pooling. In practice, it is common to see partial restrictions on risk classification, where particular risk factors such as gender or genetic test results or family history are banned. Our model does not explicitly consider such scenarios. However, we note that in our model, loss coverage is maximised when there is an intermediate level of adverse selection, not too low and not too high. It is possible that in some markets, complete pooling generates too much adverse selection; but partial restrictions on risk classification generate an intermediate level of adverse selection, and hence higher loss coverage than either pooling or fully risk-differentiated premiums.

Thus from a public policy perspective, the concept of loss coverage offers a possible rationale for some degree of restriction on risk classification. Loss coverage also provides a metric for assessing in particular cases whether the degree of restriction produces a good or bad result for the population as a whole. Insurers typically take a different view, arguing against any and all restrictions on risk classification. However, note that from the insurance industry's perspective, maximising loss coverage is equivalent to maximising premium income. Our model assumes that insurers make zero profits in equilibrium under all risk classification schemes, but in practice insurers hope to earn profits. If these profits are proportional to premiums, restrictions on risk classification which maximise loss coverage could be advantageous to the insurance industry.

8. Conclusions

This paper has investigated insurance market equilibrium under restricted risk classification with iso-elastic demand. The equilibrium was characterised by three quantities: equilibrium premium, adverse selection, and loss coverage, defined as the expected losses compensated by insurance. We investigated how these quantities varied depending on the elasticity of demand for insurance, which was assumed to be equal for high and low risk-groups.

The equilibrium premium (and adverse selection) increases monotonically with demand elasticity. However, loss coverage ratio increases from 1, to a maximum for demand elasticity of around 0.5 and then decreases, falling back to 1 for demand elasticity of 1. So, restricting risk classification increases loss coverage if demand elasticity is less than 1. This is despite the fact that restricting risk classification will always increase adverse selection. In other words, the concept of loss coverage suggests that adverse selection is not always a bad thing.

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Appendix

Result 1: For $\lambda > 0$,

$$\lambda \lesseqgtr 1 \Rightarrow C(\lambda) \gtreqless 1. \tag{27}$$

Proof. The loss coverage ratio for the case of equal demand elasticity is given in Equation 23 and can be expressed as follows:

$$C(\lambda) = \frac{1}{\pi_0^\lambda} \frac{\alpha_1\mu_1^{\lambda+1} + \alpha_2\mu_2^{\lambda+1}}{\alpha_1\mu_1 + \alpha_2\mu_2}, \quad \text{where} \quad \pi_0 = \frac{\alpha_1\mu_1^{\lambda+1} + \alpha_2\mu_2^{\lambda+1}}{\alpha_1\mu_1^\lambda + \alpha_2\mu_2^\lambda}; \tag{28}$$

$$= \left[w\mu_1^{\lambda-1} + (1-w)\mu_2^{\lambda-1}\right]^\lambda \left[w\mu_1^\lambda + (1-w)\mu_2^\lambda\right]^{1-\lambda} \quad \text{where} \quad w = \frac{\alpha_1\mu_1}{\alpha_1\mu_1 + \alpha_2\mu_2}; \tag{29}$$

$$= E_w\left[\mu^{\lambda-1}\right]^\lambda E_w\left[\mu^\lambda\right]^{1-\lambda}, \tag{30}$$

where E_w denotes expectation in this context and the random variable μ takes values μ_1 and μ_2 with probabilities w and $1-w$ respectively.

Case $\lambda = 1$**:** It follows directly from Equation 30 that $C(1) = 1$.

Case $0 < \lambda < 1$**:** Holder's inequality states that, if $1 < p, q < \infty$ where $1/p + 1/q = 1$, for positive random variables X, Y with $E[X]^p, E[Y]^q < \infty$, $E\left[X^p\right]^{1/p} E\left[Y^q\right]^{1/q} \geq E[XY]$.

Setting $1/p = \lambda$, $1/q = 1-\lambda$, $X = \mu^{\lambda(\lambda-1)}$ and $Y = 1/X$, applying Holder's inequality on Equation 30 gives,

$$C(\lambda) = E_w\left[X^{1/\lambda}\right]^\lambda E_w\left[Y^{1/(1-\lambda)}\right]^{1-\lambda} \geq E_w[XY] = 1. \tag{31}$$

Case $\lambda > 1$**:** Lyapunov's inequality states that, for positive random variable μ and $0 < s < t$, $E[\mu^t]^{1/t} \geq E[\mu^s]^{1/s}$.

So Equation 30 gives:

$$C(\lambda) = \frac{E_w\left[\mu^{\lambda-1}\right]^\lambda}{E_w\left[\mu^\lambda\right]^{\lambda-1}} = \left[\frac{E_w\left[\mu^{\lambda-1}\right]^{1/(\lambda-1)}}{E_w\left[\mu^\lambda\right]^{1/\lambda}}\right]^{\lambda(\lambda-1)} \leq 1, \tag{32}$$

as $E_w\left[\mu^{\lambda-1}\right]^{1/(\lambda-1)} \leq E_w\left[\mu^\lambda\right]^{1/\lambda}$ for $\lambda > 1$ by Lyapunov's inequality.

∎

Result 2: For $0 < \lambda < 1$,

$$\max_{w,\lambda} C = \frac{1}{2}\left(\sqrt[4]{\beta} + \frac{1}{\sqrt[4]{\beta}}\right). \tag{33}$$

Firstly, we can prove that

For $0 < \lambda < 1$,

$$\max_{w} C(\lambda) = \frac{\beta - 1}{\beta^{\lambda(1-\lambda)}\left(\frac{\beta^{\lambda}-1}{\lambda}\right)^{\lambda}\left(\frac{\beta^{1-\lambda}-1}{1-\lambda}\right)^{1-\lambda}}, \quad \text{where} \quad \beta = \frac{\mu_2}{\mu_1} > 1. \tag{34}$$

Proof. Proceeding from Equation 29, we have:

$$C(\lambda) = \frac{\left[w\beta^{1-\lambda} + (1-w)\right]^{\lambda}\left[w + (1-w)\beta^{\lambda}\right]^{1-\lambda}}{\beta^{\lambda(1-\lambda)}} \tag{35}$$

$$\Rightarrow \frac{\partial}{\partial w}\log C(\lambda) = \frac{\lambda(\beta^{1-\lambda}-1)}{w\beta^{1-\lambda} + (1-w)} - \frac{(1-\lambda)(\beta^{\lambda}-1)}{w + (1-w)\beta^{\lambda}} \tag{36}$$

$$\Rightarrow \frac{\partial^2}{\partial w^2}\log C(\lambda) = -\frac{\lambda(\beta^{1-\lambda}-1)^2}{\left[w\beta^{1-\lambda} + (1-w)\right]^2} - \frac{(1-\lambda)^2(\beta^{\lambda}-1)^2}{\left[w + (1-w)\beta^{\lambda}\right]^2} < 0. \tag{37}$$

$$\Rightarrow \frac{\partial}{\partial w}\log C(\lambda) = 0 \tag{38}$$

$$\Rightarrow w = \frac{\lambda(\beta - 1) - (\beta^{\lambda} - 1)}{(\beta^{1-\lambda} - 1)(\beta^{1-\lambda} - 1)} \quad \text{gives the maximum.} \tag{39}$$

Inserting the value of w in Equation 35, gives the required result.

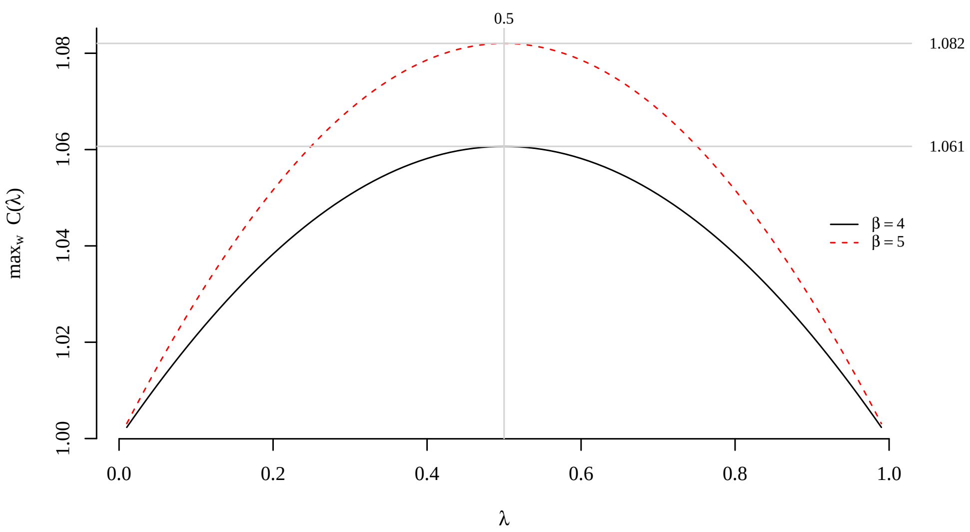

Figure 4: Maximum loss coverage ratio as a function of λ for specific values of β.

Figure 4 shows the plots of $\max_w C(\lambda)$ for $\beta = 4, 5$.

Equation 34 can also be expressed as:

$$\max_w C(\lambda) = \frac{1}{2}\left(\sqrt[4]{\beta} + \frac{1}{\sqrt[4]{\beta}}\right) \frac{2\left(\sqrt[4]{\beta} - \frac{1}{\sqrt[4]{\beta}}\right)}{\left(\frac{\left(\sqrt{\beta}\right)^{\lambda} - \frac{1}{\left(\sqrt{\beta}\right)^{\lambda}}}{\lambda}\right)^{\lambda} \left(\frac{\left(\sqrt{\beta}\right)^{1-\lambda} - \frac{1}{\left(\sqrt{\beta}\right)^{1-\lambda}}}{1-\lambda}\right)^{1-\lambda}}, \tag{40}$$

$$= \frac{1}{2}\left(\sqrt[4]{\beta} + \frac{1}{\sqrt[4]{\beta}}\right) \frac{R(1/2)}{R(\lambda)}, \tag{41}$$

$$\text{where} \quad R(\lambda) = \left(\frac{\left(\sqrt{\beta}\right)^{\lambda} - \frac{1}{\left(\sqrt{\beta}\right)^{\lambda}}}{\lambda}\right)^{\lambda} \left(\frac{\left(\sqrt{\beta}\right)^{1-\lambda} - \frac{1}{\left(\sqrt{\beta}\right)^{1-\lambda}}}{1-\lambda}\right)^{1-\lambda}. \tag{42}$$

The result follows from $R(\lambda) \geq R(1/2)$, which in turn follows from the fact that $R(\lambda)$ is symmetric and convex over $0 < \lambda < 1$. As symmetry is obvious, we only need to prove convexity of $R(\lambda)$.

Note that,

$$\log R(\lambda) = g(\lambda) + g(1-\lambda), \quad \text{where} \quad g(\lambda) = \lambda \log\left(\frac{\left(\sqrt{\beta}\right)^{\lambda} - \frac{1}{\left(\sqrt{\beta}\right)^{\lambda}}}{\lambda}\right). \tag{43}$$

If $g(\lambda)$ is a convex function over (0,1), then $g''(\lambda) \geq 0$ and $g''(1-\lambda) \geq 0$, so $\log R(\lambda)$ is convex, which in turn implies $R(\lambda)$ is convex. So it suffices to show that:

$$g(x) = x \log\left(\frac{a^x - a^{-x}}{x}\right) \tag{44}$$

is convex over (0,1), where $a = \sqrt{\beta} > 1$. Now,

$$g'(x) = \log\left(\frac{a^x - a^{-x}}{x}\right) + \left(\frac{a^x + a^{-x}}{a^x - a^{-x}}\right) x \log a - 1. \tag{45}$$

$$g''(x) = \frac{(a^x + a^{-x})x \log a - (a^x - a^{-x})}{x(a^x - a^{-x})} + \frac{a^{2x} - a^{-2x} - 4x \log a}{(a^x - a^{-x})^2} \log a \geq 0, \tag{46}$$

as both $[(a^x + a^{-x})x \log a - (a^x - a^{-x})]$ and $[a^{2x} - a^{-2x} - 4x \log a]$ are increasing functions starting from 0 at $x = 0$. Hence proved. ■