INSURANCE LOSS COVERAGE AND SOCIAL WELFARE

MingJie Hao, Angus S. Macdonald, Pradip Tapadar and R. Guy Thomas*

ABSTRACT

Restrictions on insurance risk classification may induce adverse selection, which is usually perceived to reduce efficiency. We suggest a counter-argument to this perception in circumstances where modest adverse selection leads to an increase in 'loss coverage', defined as the expected losses compensated by insurance for the whole population. This happens if the shift in coverage towards higher risks under adverse selection more than outweighs the fall in number of individuals insured. We also reconcile the concept of 'loss coverage' and a utilitarian concept of social welfare. For iso-elastic insurance demand, ranking risk classification schemes by (observable) loss coverage always gives the same ordering as ranking by (unobservable) utilitarian social welfare.

KEYWORDS

Adverse selection; loss coverage; social welfare. J.E.L. Classification: D82, G22.

*Hao: School of Mathematics, Statistics and Actuarial Science, University of Kent, Canterbury, CT2 7NF, UK. Macdonald: Department of Actuarial Mathematics and Statistics, and the Maxwell Institute for Mathematical Sciences, Heriot-Watt University, Edinburgh EH14 4AS, UK. Tapadar: School of Mathematics, Statistics and Actuarial Science, University of Kent, Canterbury, CT2 7NF, UK. Corresponding author: Pradip Tapadar, School of Mathematics, Statistics and Actuarial Science, University of Kent, Canterbury, CT2 7NF, UK. Email: P.Tapadar@kent.ac.uk, Phone: +44 1227 824169, Fax: +44 1227 827932. Thomas: School of Mathematics, Statistics and Actuarial Science, University of Kent, Canterbury, CT2 7NF, UK.

1 Introduction

Restrictions on insurance risk classification are pervasive in life insurance and other personal insurance markets. For example, gender classification in insurance pricing has been banned in the European Union since 2012; in the US, the Patient Protection and Affordable Care Act allows classification only by age, location, family size and smoking status; and many countries have restricted insurers' use of genetic test results. Such restrictions may increase equity, but they also create asymmetries in (the use of) information, and hence are usually seen by economists as reducing efficiency.

A simple version of the usual efficiency argument is as follows. If insurers are not permitted to charge risk-differentiated prices, they have to pool different risks at a common pooled price.[1] This pooled price is cheap for higher risks and expensive for lower risks; so more insurance is bought by higher risks, and less insurance is bought by lower risks. The equilibrium pooled price of insurance is higher than a population-weighted average of true risk premiums. Also, in most markets the number of higher risks is smaller than the number of lower risks, so the total number of risks insured falls. The usual efficiency argument focuses on this reduction in coverage, e.g. "This reduced pool of insured individuals reflects a decrease in the efficiency of the insurance market" (Dionne and Rothschild [2014, p185]).

[1] In this article we disregard the possibility that insurers banned from classifying risks induce separation of risk-groups by menus of contracts offering different levels of cover priced at different rates (e.g. Rothschild and Stiglitz [1976]). Our reasons are explained in the literature review section.

In an earlier article, Thomas (2008) suggested that in some circumstances, there is a counter-argument to this perception of reduced efficiency. The rise in equilibrium price under pooling reflects a shift in coverage towards higher risks. If the shift in coverage is large enough, it can more than outweigh the fall in numbers insured. In these circumstances, despite fewer risks being insured under pooling, expected losses compensated by insurance – 'loss coverage' – can be higher. Since more risk is being voluntarily traded and more losses are being compensated, this might be regarded as a better outcome from pooling.

The argument just given rests on the general intuition that more losses being compensated is a 'good thing' . The relationship of loss coverage to any formal concept of social welfare is not immediately apparent, and was not addressed in Thomas (2008). The present article reconciles the concept of loss coverage to a utilitarian concept of social welfare. Specifically, we show that if insurance demand is iso-elastic, the risk classification scheme which maximises loss coverage also maximises social welfare. From a utilitarian policymaker's perspective, this may be a useful result, because maximising loss coverage does not require knowledge of individuals' (generally unobservable) utility functions; loss coverage is based solely on observable quantities.

2 Motivating example

The possibility that loss coverage may be increased by restrictions on risk classification can be illustrated by heuristic examples of insurance market equilibria under two alternative risk classification regimes: actuarially fair premiums and pooled premiums.

Suppose that in a population of 2,000 risks, 32 losses are expected every year. There are two risk-groups. The high risk-group of 400 individuals has a probability of loss 4 times higher than those in the low risk-group. This is summarised in Table 1.

We assume that probability of loss is not altered by the purchase of insurance, i.e. there is no moral hazard. An individual's risk-group is fully observable to insurers and all insurers are required to use the same risk classification regime. The equilibrium price of insurance is determined as the price at which insurers make zero profit.

Under the first risk classification regime, insurers charge premiums which are actuarially fair to members of each risk-group. The proportion of each risk-group which buys insurance under these conditions, i.e. the 'fair-premium proportional demand', is 50%, in line with industry statistics. Table 1 shows the outcome. Half the losses in the population are compensated by insurance. We heuristically characterise this as a 'loss coverage' of 0.5.

Now suppose that a new risk classification regime is introduced, where insurers have to charge a single 'pooled' price to members of both the low

Table 1: Equilibrium under actuarially fair premiums: lower loss coverage.

	Risk-group		
	Low risk	High risk	Aggregate
Risk	0.01	0.04	0.016
Total population	1600	400	2,000
Expected population losses	16	16	32
Break-even premiums (differentiated)	0.01	0.04	0.016
Numbers insured	800	200	1,000
Insured losses	8	8	16
Loss coverage			0.5

and high risk-groups. One possible outcome is shown in Table 2, which can be summarised as follows:

(a) The pooled premium of 0.0194 at which insurers make zero profits is calculated as the demand-weighted average of the risk premiums: (600 x 0.01 + 275 x 0.04)/875 = 0.0194).

(b) The pooled premium is expensive for low risks, so fewer of them buy insurance (600, compared with 800 before). The pooled premium is cheap for high risks, so more of them buy insurance (275, compared with 200 before). Because there are 4 times as many low risks as high risks in the population, the total number of policies sold falls (875, compared with 1,000 before).

(c) The resulting loss coverage is 0.53125. The shift in coverage towards high risks more than outweighs the fall in number of policies sold: 17 of

Table 2: Equilibrium under pooled premiums: higher loss coverage.

	Risk-group		
	Low risk	High risk	Aggregate
Risk	0.01	0.04	0.016
Total population	1600	400	2000
Expected population losses	16	16	32
Break-even premiums (pooled)	0.0194	0.0194	0.0194
Numbers insured	600	275	875
Insured losses	6	11	17
Loss coverage			0.53125

the 32 losses (53%) in the population as a whole are now compensated by insurance (compared with 16 of 32 before).

The occurrence of the favourable outcome (higher loss coverage) under asymmetric information and pooling in Table 2 depends on the demand elasticities for insurance in high and low risk groups. Later in this article, we will show that the required demand elasticities are plausible.

3 Literature Review

The model of insurance markets implied by the heuristic example above differs from canonical models derived from Rothschild and Stiglitz [1976] in two main ways.

First, in our model insurers compete only on price; they do not induce separation of risk-groups by menus of contracts offering different levels of cover priced at different rates. In this respect, our model is more in the spirit of Akerlof [1970]. We justify this approach by noting that some important markets, such as life insurance, have non-exclusive contracting, and so separation via contract menus is not feasible. Furthermore, as far as we are aware, the concept of separation via contract menus is also not salient to practitioners in other markets where restrictions on risk classification apply, for example auto insurance in the European Union.[2]

Second, in our model agents with identical probabilities of loss can have different utility functions, and so unlike the representative agents from each risk-group in Rothschild-Stiglitz type models, they do not all make the same purchasing decision. This leads to an equilibrium where not all agents are insured; this corresponds to the empirical reality of most voluntary insurance markets.[3]

Previous articles on loss coverage (Hao et al. [2016], Thomas [2008, 2009]) formalised the heuristic example above by a model with two risk-groups with

[2]As regards life insurance, Rothschild-Stiglitz type models are inconsistent in important ways with empirical data (e.g. Cawley and Philipson [1999]). As regards practice in other insurance markets, most actuarial pricing textbooks make no reference whatsoever to the concept of menus of contracts as screening devices (e.g. Gray and Pitts [2012], Friedland [2013], Parodi [2014]). Other actuarial textbooks specifically counsel *against* any thought of using the level of deductible as a pricing factor (e.g. Ohlsson and Johansson [2010]).

[3]For example, in life insurance, the Life Insurance Market Research Organisation (LIMRA) states that 44% of US households have some individual life insurance (LIMRA [2013]). The American Council of Life Insurers states that 144m individual policies were in force in 2013 (American Council of Life Insurers [2014, p72]); the US adult population (aged 18 years and over) at 1 July 2013 as estimated by the US Census Bureau was 244m. In health insurance, only 14.6% of the US population has individually purchased private cover (US Census Bureau, 2015), albeit substantially more have employer group cover or Medicare or Medicaid government cover.

higher and lower probabilities of loss. Insurance demand from each risk-group at each price was modelled by a demand function with output a number between 0 and 1, to reflect the empirical observation that not all individuals buy insurance at each price. The variation in purchasing decisions across persons within each risk-group (i.e. with the same probabilities of loss) was characterised as stochastic; no reference was made to individual utilities.

The loss coverage literature contrasts with economic literature on insurance risk classification, as summarised in surveys such as Hoy [2006], Einav and Finkelstein [2011] and Dionne and Rothschild [2014]. Economic literature typically takes a utility-based approach: representative agents from each risk-group make purchasing decisions which maximise their expected utilities, and the outcomes of different risk classification schemes are then evaluated by a social welfare function which is a (possibly weighted) sum of expected utilities over the whole population. For example Hoy [2006] uses a utilitarian social welfare function which assigns equal weights to the utilities of all individuals. Einav and Finkelstein [2011] use a deadweight-loss concept which appears equivalent to a social welfare function with utilities cardinalized so as to weight willingness-to-pay equally across all individuals.

The present article connects the loss coverage literature with the economic literature in two ways. First, we provide a utility-based micro-foundation for the proportional insurance demand function, driven by variations between individuals in their utility functions, which can explain why only a proportion of the individuals in each risk-group buy insurance at each price. Second, we reconcile loss coverage to the utilitarian concept of social welfare described above. Specifically, we show that if insurance demand is iso-elastic, the risk classification scheme which maximises loss coverage also maximises social welfare.

4 Insurance Demand for a Single Risk-group

4.1 Utility of Wealth and Certainty Equivalence

Consider an individual with an initial wealth W, who is exposed to the risk of losing an amount of L with probability μ. Suppose preference for wealth is driven by the utility function $U(w)$, which is increasing in wealth w, i.e. $U'(w) > 0$. Individuals are typically also assumed to be risk-averse i.e. $U''(w) < 0$, but our theory of insurance demand does *not* require that *all* individuals are risk-averse.

Suppose that the individual is offered insurance against the full amount of loss L at premium rate π per unit of loss, i.e. for premium πL. She will choose to buy insurance if π is low enough to satisfy:

$$U(W - \pi L) > (1 - \mu)U(W) + \mu\, U(W - L). \qquad (1)$$

In the above model, all individuals with the same utility function and probability of loss either buy insurance or they do not, based on whether or not the premium being charged, π, is low enough. However, in real insurance markets, we typically observe that not all individuals with the same probability of loss make the same purchasing decision (e.g. for life insurance, see the figures in footnote 3). How can this variation in insurance purchasing decisions be explained?

One plausible explanation suggested by a number of authors (e.g. Finkelstein and McGarry [2006]; Cutler et al. [2008]) is that risk preferences vary between individuals. To formulate this variability, let us assume a population of individuals, all with the same risk μ but who may have different utility functions. Suppose for simplicity that utility functions belong to a family parameterized by a positive real number γ. So a particular individual's utility function can be denoted by $U_\gamma(w)$.

Further suppose that an individual's utility function parameter γ is sampled randomly from an underlying random variable Γ with distribution function $F_\Gamma(\gamma)$. So, a particular individual's utility function, $U_\gamma(w)$, is a random quantity, the randomness being induced by $F_\Gamma(\gamma)$.

Based on this formulation, an individual will choose to buy insurance if and only if the following condition is satisfied for the combination of the offered premium π and their particular utility function $U_\gamma(w)$:

$$U_\gamma(W - \pi L) > (1 - \mu)U_\gamma(W) + \mu\, U_\gamma(W - L), \tag{2}$$

Note that all individuals are behaving deterministically, given their knowledge.

Although utility functions of different individuals can have different origins and scales, certainty-equivalent decisions are independent of these choices. So without loss of generality, we will assume that all individuals have the same utility at the "end points" $W - L$ and W. And for clarity, we will suppress the subscript γ for the utility at the "end points" and write $U(W)$ and $U(W - L)$ as they are the same for all individuals. We can then write Equation (2) as:

$$U_\gamma(W - \pi L) > u_c \text{ where} \tag{3}$$

$$u_c = (1 - \mu)U(W) + \mu\, U(W - L) \text{ is a constant.} \tag{4}$$

This says that an individual insures if the utility from insurance exceeds a critical value u_c. Note that u_c is the same for all individuals who are exposed to the same probability of loss.

Figure 1 provides a graphical representation showing utility functions of four individuals with the same probability of loss μ. The concave utility curves, with points A, B and C, represent risk-averse individuals, where higher concavity represents higher risk-aversion. We also show a convex utility curve, with point D, which represents a risk-loving (or risk-neglecting) individual. (As mentioned previously, the model does not require that all individuals are risk-averse.) For the individual at point A, the utility with insurance, $U_{\gamma_A}(W - \pi L)$, exceeds the critical value u_c, where γ_A is the individual's utility function parameter. So the individual buys insurance. For the

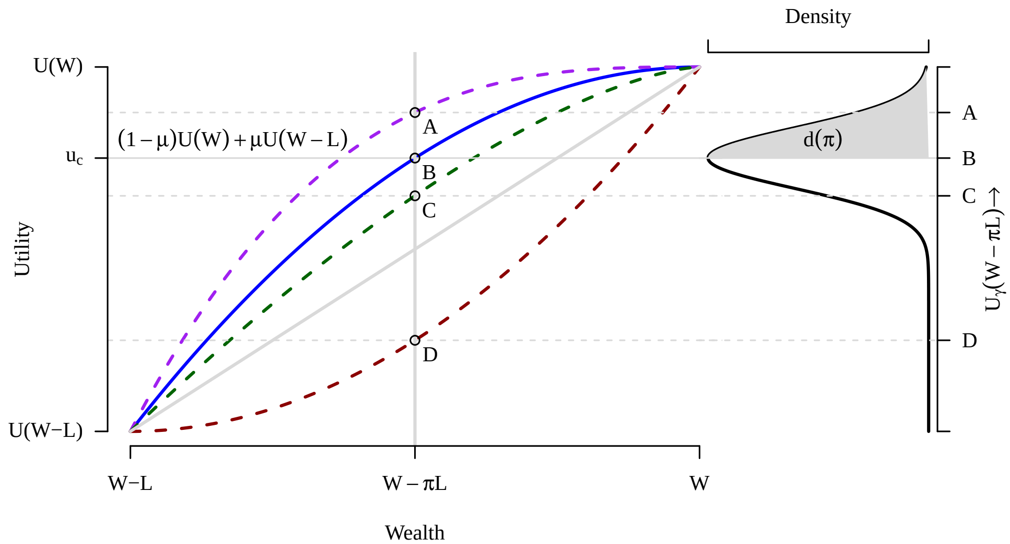

Figure 1: Heterogeneous utility functions within a risk-group, leading to proportional insurance demand.

individuals at points C and D, the inverse applies, so they do not purchase insurance. The individual at point B is indifferent.

The utility at the fixed wealth $(W - \pi L)$ is a random variable, that we denote by $U_\Gamma(W - \pi L)$. The distribution function of $U_\Gamma(W - \pi L)$ is induced by that of Γ and we denote it by $G_\Gamma(\gamma)$. The corresponding probability density function of the utilities at that level of wealth is shown in the rotated plot on the right-hand side of Figure 1.

Now assume that the insurer cannot observe individuals' utility functions. Then, for given offered premium π, all the insurer can observe of insurance purchasing behaviour is the proportion of individuals who buy insurance. We call this a demand function and denote it by $d(\pi)$. We have:

$$d(\pi) = \mathrm{P}\left[U_\Gamma(W - \pi L) > u_c\right] = 1 - G_\Gamma(u_c). \tag{5}$$

Insurance purchase is denoted by the shaded area, $d(\pi)$, under the density graph for $U_\Gamma(W - \pi L)$.

We note the following three properties of demand for insurance:

(a) $0 \leq d(\pi) \leq 1$, so $d(\pi)$ is a valid probability.

(b) $d(\pi)$ is non-increasing in π, i.e. demand for insurance cannot increase when premium increases. This can be shown as follows: For utility functions with $U'(w) > 0$, if $\pi_1 < \pi_2$, the random variable $U_\Gamma(W - \pi_1 L)$ is statewise dominant[4] over the random variable $U_\Gamma(W - \pi_2 L)$. So,

$$\begin{aligned} \pi_1 < \pi_2 &\Rightarrow \mathrm{P}\left[U_\Gamma(W - \pi_1 L) > u_c\right] \geq \mathrm{P}\left[U_\Gamma(W - \pi_2 L) > u_c\right] \\ &\Rightarrow d(\pi_1) \geq d(\pi_2). \end{aligned} \tag{6}$$

(c) Each individual's decision is completely deterministic, given what they know. But to the insurer it appears stochastic, given what the insurer knows.

As noted earlier, certainty equivalent decisions do not depend on the origins and scales of utility functions, so we can standardise the utility functions

[4] One random variable is statewise dominant over a second if the first is at least as high as the second in all states of nature, with strict inequality for at least one state. It is an absolute form of dominance.

such that all individuals have the same utilities $U(W)$ and $U(W-L)$ at the "end points" W and $W-L$. The following standardisation is convenient:

$$U(W) = 1, \tag{7}$$

$$U(W-L) = 0. \tag{8}$$

The constant u_c in Equation (5) then becomes $(1-\mu)$, and so the demand for insurance is:

$$d(\pi) = \mathrm{P}[U_\Gamma(W - \pi\, L) > 1 - \mu]. \tag{9}$$

4.2 Iso-elastic Demand

This sub-section gives an illustrative example of the link from a specific distribution of risk preferences to a specific proportional demand for insurance where individuals are exposed to the same probability of loss.

Suppose $W = L = 1$ with a power utility function:

$$U_\gamma(w) = w^\gamma, \tag{10}$$

so that $U_\gamma(0) = 0$ and $U_\gamma(1) = 1$. This particular form of utility function leads to:

$$\text{relative risk aversion coefficient:} \;\; -w\frac{U''_\gamma(w)}{U'_\gamma(w)} = 1 - \gamma. \tag{11}$$

So the heterogeneity in preferences between individuals can be modelled through the randomness of the risk aversion parameter γ. As outlined in Section 4.1, we define a positive random variable Γ, and individual risk preferences γ are then instances drawn from the distribution of Γ.

Demand for insurance at a given premium π is then:

$$
\begin{aligned}
d(\pi) &= \mathrm{P}\left[U_\Gamma(1-\pi) > 1-\mu\right], && (12)\\
&= \mathrm{P}\left[(1-\pi)^\Gamma > 1-\mu\right], && (13)\\
&= \mathrm{P}\left[\Gamma \log(1-\pi) > \log(1-\mu)\right], \text{ as log is monotonic,} && (14)\\
&= \mathrm{P}\left[\Gamma < \frac{\log(1-\mu)}{\log(1-\pi)}\right], \text{ as } \log(1-\pi) < 0, && (15)\\
&\approx \mathrm{P}\left[\Gamma < \frac{\mu}{\pi}\right], \text{ as } \log(1-x) \approx -x, \text{ for small } x. && (16)
\end{aligned}
$$

Now suppose Γ has the following distribution:

$$
F_\Gamma(\gamma) = \mathrm{P}\left[\Gamma \le \gamma\right] = \begin{cases} 0 & \text{if } \gamma < 0 \\ \tau\,\gamma^\lambda & \text{if } 0 \le \gamma \le (1/\tau)^{1/\lambda} \\ 1 & \text{if } \gamma > (1/\tau)^{1/\lambda}, \end{cases} \tag{17}
$$

where τ and λ are positive parameters. Note that $\tau = \lambda = 1$ leads to a uniform distribution. λ controls the shape of the distribution function and τ controls the range over which Γ takes its values.[5]

Based on this distribution for Γ, the demand for insurance in Equation

[5] This is a generalised version of the Kumaraswamy distribution, which in its standard form takes values only over [0,1] (Kumaraswamy [1980]).

(16) takes the form:

$$d(\pi) = \tau \left(\frac{\mu}{\pi}\right)^{\lambda}, \tag{18}$$

subject to a cap of 1 (when all members of a risk-group purchase insurance, demand cannot increase further). This corresponds to iso-elastic demand, the constant demand elasticity being:

$$\epsilon(\pi) = -\frac{\partial \log(d(\pi))}{\partial \log \pi} = \lambda. \tag{19}$$

The parameter τ can also be interpreted as the *fair-premium demand*, that is the demand when an actuarially fair premium is charged.

The motivating example given earlier can be shown to correspond to this iso-elastic demand function, with fair-premium demand $\tau = 0.5$ and constant demand elasticity $\lambda = 0.435$ for both risk-groups. These are reasonable parameters.[6]

An important point to note here is that power utility function of the form given in Equation (10) is concave only if the risk aversion parameter γ is less than 1. Such a constraint can be imposed on random variable Γ by setting $\tau = 1$ in Equation (17). Then the third branch of Equation (17) implies that $d(\pi) = 1$ for $\pi < \mu$, which corresponds to the standard assumption in the economics literature that all individuals are risk-averse and hence will buy

[6] Approximately half the population has some life insurance (see footnote 3). For yearly renewable term insurance in the US, demand elasticity has been estimated at 0.4 to 0.5 (Pauly et al. [2003]). A questionnaire survey about life insurance purchasing decisions produced an estimate of 0.66 (Viswanathan et al. [2006]).

insurance for premiums not exceeding their probability of loss. By permitting some individuals to be ‘risk-lovers’, the model better represents the partial take-up of insurance which is observed in practice. Although ‘risk-loving’ or ‘risk-seeking’ are the usual descriptions, ‘risk-neglecting’ might be a more realistic one.

5 Equilibrium and Loss Coverage for Two or More Risk-groups

5.1 Equilibrium

The demand model in the previous section provides an explanation of the proportional insurance demand which is observed in the real world, predicated on different risk preferences of individuals who all have the same probabilites of loss. In practice, individuals can have different probabilities of loss (i.e. different risk-groups). The present section considers insurance market equilibrium for two or more risk-groups.

For simplicity, we assume all wealth and losses are of unit amount, that is $W = L = 1$. Suppose the population can be sub-divided into n distinct risk-groups with probabilities of loss given by $\mu_1, \mu_2, \ldots, \mu_n$. For convenience, we assume $0 < \mu_1 < \mu_2 < \ldots < \mu_n < 1$.

Suppose the proportion of the population belonging to risk-group i is p_i, for $i = 1, 2, \ldots, n$. If we choose an individual at random from the population, their probability of loss is a random variable, which we denote by μ, and its distribution is given by $\mathrm{P}[\mu = \mu_i] = p_i$ for $i = 1, 2, \ldots, n$.

Suppose insurers charge premiums $\pi_1, \pi_2, \ldots, \pi_n$ for the risk-groups $i = 1, 2, \ldots, n$, respectively. Based on the model of Section 4, the demand for insurance within risk-group i is denoted by $d_i(\pi_i)$, where $0 \leq d_i(\pi_i) \leq 1$ and $d_i(\pi_i)$ is non-increasing in π_i.

Let the insurance purchasing decision of an individual chosen at random from the whole population be represented by the indicator random variable Q, taking the value of 1 if insurance is purchased; and 0 otherwise. Within risk-group i, Q is a Bernoulli random variable defined by:

$$\mathrm{E}[\, Q \mid \mu = \mu_i \,] = \mathrm{P}[\, Q = 1 \mid \mu = \mu_i \,] = d_i(\pi_i). \tag{20}$$

Then the expected demand for insurance across the the whole population, i.e. the expected proportion who buy insurance, is the unconditional expected value of Q:

$$\mathrm{E}[\, Q \,] = \sum_{i=1}^{n} \mathrm{E}[\, Q \mid \mu = \mu_i \,]\, \mathrm{P}[\mu = \mu_i] = \sum_{i=1}^{n} d_i(\pi_i) p_i. \tag{21}$$

Now suppose that the occurrence of a loss event for an individual chosen at random from the whole population is represented by the indicator random variable, X, taking the value of 1 if a loss event has occurred; and 0 otherwise.

Within risk-group i, the loss X is a Bernoulli random variable defined as:

$$\mathrm{E}[\,X \mid \mu = \mu_i\,] = \mathrm{P}[\,X = 1 \mid \mu = \mu_i\,] = \mu_i. \tag{22}$$

Then the expected population loss is the unconditional expected value of X:

$$\mathrm{E}[\,X\,] = \sum_{i=1}^{n} \mathrm{E}[\,X \mid \mu = \mu_i\,]\,\mathrm{P}[\mu = \mu_i] = \sum_{i=1}^{n} \mu_i p_i. \tag{23}$$

Conditional on $\mu = \mu_i$, we assume that Q and X are independent. This ensures that there is no moral hazard; although the level of risk may influence the decision to buy insurance, mediated by $d_i(\pi_i)$, insured individuals in any risk-group have the same probability of loss as uninsured individuals. The expected insurance claim in respect of an individual chosen at random from the population is then:

$$\begin{aligned}
\mathrm{E}[\,QX\,] &= \sum_{i=1}^{n} \mathrm{E}[\,QX \mid \mu = \mu_i\,]\,\mathrm{P}[\mu = \mu_i], \\
&= \sum_{i=1}^{n} \mathrm{E}[\,Q \mid \mu = \mu_i\,]\,\mathrm{E}[\,X \mid \mu = \mu_i\,]\,\mathrm{P}[\mu = \mu_i], \\
&= \sum_{i=1}^{n} d_i(\pi_i)\mu_i p_i. \qquad (24)
\end{aligned}$$

Next, define Π to be the premium paid by an individual chosen at random from the population. Then Π is a random variable. Since individuals who do not purchase insurance pay a premium of zero, we have $\Pi = Q\Pi$. Then within risk-group i:

$$\mathrm{E}[\,\Pi \mid \mu = \mu_i\,] = \mathrm{E}[\,Q\Pi \mid \mu = \mu_i\,] = \mathrm{E}[\,Q \mid \mu = \mu_i\,]\pi_i = d_i(\pi_i)\pi_i. \qquad (25)$$

Then the unconditional expected premium income is:

$$\mathrm{E}[\,\Pi\,] = \sum_{i=1}^{n} \mathrm{E}[\,\Pi \mid \mu = \mu_i\,]\,\mathrm{P}[\mu = \mu_i] = \sum_{i=1}^{n} d_i(\pi_i)\pi_i p_i. \qquad (26)$$

The expected profit for insurers, as a function of the risk classification regime $\underline{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$, is then :

$$\rho(\underline{\pi}) = E[\Pi] - E[QX] = \sum_{i=1}^{n} d_i(\pi_i)\pi_i p_i - \sum_{i=1}^{n} d_i(\pi_i)\mu_i p_i. \qquad (27)$$

If this expected profit is zero, the risk classification regime satisfies the *equilibrium condition*:

$$\rho(\underline{\pi}) = 0 \;\Leftrightarrow\; \sum_{i=1}^{n} d_i(\pi_i)\pi_i p_i - \sum_{i=1}^{n} d_i(\pi_i)\mu_i p_i = 0. \qquad (28)$$

We assume that competition between insurers compels all insurers to use the same equilibrium risk classification regime. Depending on applicable regulation, two polar cases of equilibrium risk classification regimes are as follows:

- Full risk classification, under which $\pi_i = \mu_i$. Here, the insurer uses the maximum possible degree of underwriting. We denote this regime by

$\underline{\mu}$.

- No risk classification, or risk pooling, under which all $\pi_i = \pi$, a constant. Here, the insurer uses the minimum possible degree of underwriting.

By considering the insurer's expected profit under risk pooling at the extrema $\pi = \mu_1$ and $\pi = \mu_n$, it is clear that there must be at least one risk pooling regime which leads to equilibrium.[7]

5.2 Loss coverage

We define loss coverage under a risk classification $\underline{\pi}$ that leads to equilibrium as the expected losses across the whole population that are compensated by insurance, i.e. $\mathrm{E}[QX]$ as defined in Equation (24). That is:

$$\text{Loss coverage: } LC(\underline{\pi}) = E[QX], \tag{29}$$

where $\underline{\pi}$ satisfies the equilibrium condition in Equation (28).

To compare the relative merits of different risk classification regimes, we need to define a reference level of loss coverage. We use the level under

[7]Uniqueness is not guaranteed, but multiple solutions do not arise for plausible demand elasticities (for details see Appendix B of Thomas [2017]). Alternatively, the lowest of any multiple roots can arguably be regarded as the only true equilibrium. This is because any putative equilibrium above the lowest root can be broken by one insurer charging slightly more than the lowest root (Hoy and Polborn [2000]).

actuarially fair premiums, and so define the *loss coverage ratio*, as follows:

$$\text{Loss coverage ratio: } C = \frac{LC(\underline{\pi})}{LC(\underline{\mu})}. \tag{30}$$

5.3 Iso-elastic demand: Equilibrium and loss coverage

We now continue the iso-elastic example developed in Section 4.2. Suppose there are n risk-groups with population proportions $p_1, p_2, \ldots, p_n$, probabilities of loss $\mu_1 < \mu_2 < \ldots < \mu_n$ and insurance demand modelled as per Equation (19):

$$d_i(\pi) = \tau_i \left(\frac{\mu_i}{\pi}\right)^{\lambda}, \quad i = 1, 2, \ldots, n. \tag{31}$$

Here, for simplicity, we are assuming that the demand elasticity of insurance λ is the same for all risk-groups.

Under this set-up, the equilibrium condition in Equation (28) requires that the premium charged $\underline{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$, satisfy:

$$\rho(\underline{\pi}) = 0 \;\Leftrightarrow\; \sum_{i=1}^{n} \tau_i \left(\frac{\mu_i}{\pi_i}\right)^{\lambda} \pi_i p_i = \sum_{i=1}^{n} \tau_i \left(\frac{\mu_i}{\pi_i}\right)^{\lambda} \mu_i p_i. \tag{32}$$

And the loss coverage at this equilibrium is given by:

$$LC(\underline{\pi}) = E[QX] = \sum_{i=1}^{n} \tau_i \left(\frac{\mu_i}{\pi_i}\right)^{\lambda} \mu_i p_i = \sum_{i=1}^{n} \frac{\mu_i^{\lambda+1}}{\pi_i^{\lambda}} \tau_i p_i. \tag{33}$$

For the special case where the same premium π_0 is charged for all risk-groups, the pooled equilibrium premium satisfying $\rho(\pi_0) = 0$ is unique and is given by:

$$\pi_0 = \frac{\sum_{i=1}^{n} \alpha_i \mu_i^{\lambda+1}}{\sum_{i=1}^{n} \alpha_i \mu_i^{\lambda}}, \quad \text{where} \quad \alpha_i = \frac{p_i \tau_i}{\sum_{j=1}^{n} p_j \tau_j}, \; i = 1, 2, \ldots, n. \tag{34}$$

that is, α_i is the *fair-premium demand-share*, that is the share of total demand represented by risk-group i when actuarially fair premiums are charged.

The loss coverage ratio, comparing loss coverage under pooled premiums to that under actuarially fair premiums, is:

$$C = \frac{1}{\pi_0^{\lambda}} \frac{\sum_{i=1}^{n} \alpha_i \mu_i^{\lambda+1}}{\sum_{i=1}^{n} \alpha_i \mu_i}, \tag{35}$$

where π_0 is the pooled equilibrium premium given in Equation (34).

Figures 2 and 3 show the plots of pooled equilibrium premium, insurance demand (cover) and loss coverage ratio as a function of demand elasticity λ, for two risk-groups where $(\mu_1, \mu_2) = (0.01, 0.04)$ and fair-premium demand-shares $(\alpha_1, \alpha_2) = (0.9, 0.1)$. Compared with the result under actuarially fair premiums, under pooling the premium is always higher, and demand (cover) is always lower. This reduction in cover is the perceived loss of efficiency arising from adverse selection. Loss coverage, on the other hand, is not always lower: for this iso-elastic demand function, it is higher than under actuarially fair premiums if demand elasticity is less than 1. There is some empirical evidence that insurance demand elasticities are typically less than 1 in many markets (Pauly et al. [2003]; Viswanathan et al. [2006]; Chernew et al. [1997]; Blumberg et al. [2001]; Buchmueller and Ohri [2006]; Butler

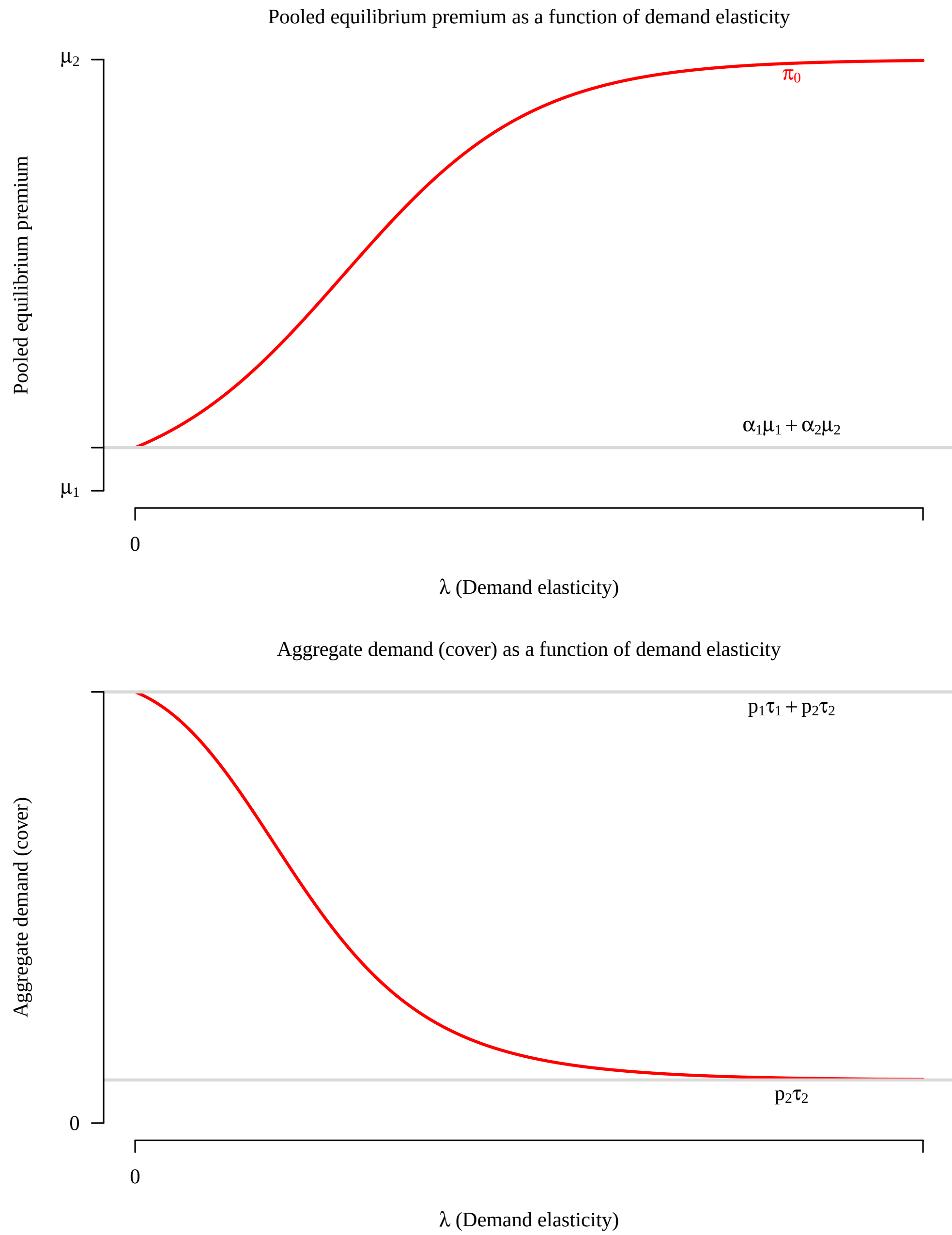

Figure 2: Pooled equilibrium premium (top panel) and aggregate demand (bottom panel) as functions of demand elasticity.

[2002]).

The pattern shown in Figure 3 is formalised by the following proposition.

Proposition 1. *If demand elasticity is a positive constant* λ *and the loss coverage ratio as defined in Equation (35) is* C*, then*

$$\lambda \lesseqgtr 1 \Leftrightarrow C \gtreqless 1 \tag{36}$$

In other words, for iso-elastic insurance demand, pooling produces higher loss coverage than actuarially fair premiums if demand elasticity is less than 1, and vice versa.

The proof of Proposition 1 is provided in Appendix A.

6 Social Welfare and Loss Coverage

6.1 Social Welfare

Our approach to social welfare is in the same spirit as Hoy [2006]: we assume cardinal and interpersonally comparable utilities, and assign equal weights to the utilities of all individuals. This equal-weights approach is based on the Harsanyi [1955] 'veil of ignorance' argument: that is, behind the (hypothetical) 'veil of ignorance', where one does not know what position in society (e.g. high risk or low risk) one occupies, the appropriate probability to assign to being any individual is $1/n$, where n is the number of individuals

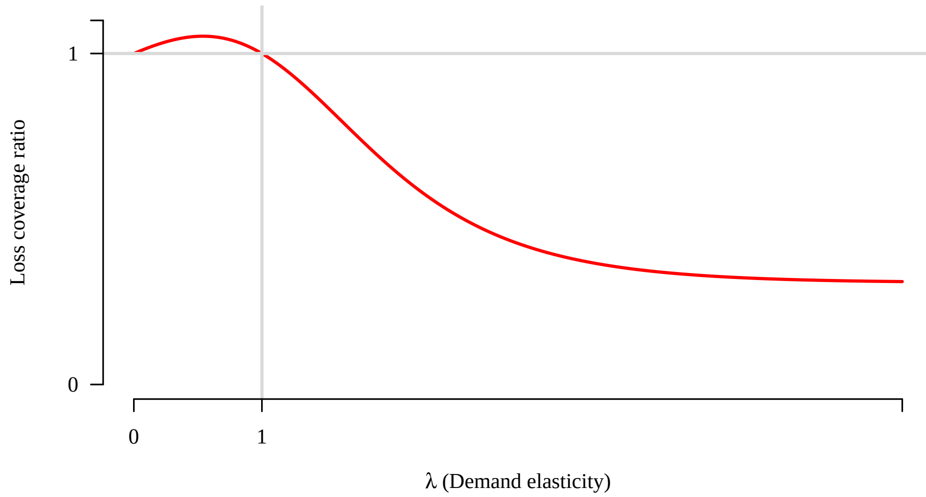

Figure 3: Loss coverage ratio as a function of demand elasticity.

in society. Alternative risk classification regimes can then be compared by comparing expected utility in each regime for the (hypothetical) individual utility-maximiser behind the 'veil of ignorance'.

In the model defined above, suppose an individual is selected at random from the whole population. The individual's expected utility can be written as follows:

$$\text{Social Welfare} \tag{37}$$
$$= \mathrm{E}\left[\, Q\, U_{\Gamma}(W - \Pi L) + (1 - Q)\left[(1 - X)\, U_{\Gamma}(W) + X\, U_{\Gamma}(W - L)\right]\right]$$

where the first part represents random utility if insurance is purchased; and the second part is the random utility if insurance is not purchased.

We assumed earlier that all individuals had the same utilities at the 'end-points', W and $W - L$. This relied on the property that certainty equivalent decisions do not depend on the origins and scales of utility functions. But this argument cannot be directly extended to Equation (37), because individuals' utilities can differ for identical levels of wealth, which has direct implications for social welfare.

However, without any standardisation, Equation (37) is susceptible to being dominated by a 'utility monster' who derives more utility from a given level of wealth than all other individuals combined (see Bailey [1997], Nozick [1974]). This makes it unsuitable for policy purposes. So we propose to continue standardising utility functions so that all utilities are the same at 'end-points', W and $W - L$, as before. This standardisation implies that the same 'disutility of uninsured loss' $[U(W) - U(W - L)]$ is assigned to all individuals, but utility if insurance is purchased $U_\Gamma(W - \Pi L)$ differs between individuals. Under this standardisation, social welfare, denoted by S can be expressed as:

$$S = \mathrm{E}\left[\, Q\, U_\Gamma(W - \Pi L) + (1 - Q)\left[(1 - X)\, U(W) + X\, U(W - L)\right]\right]. \quad (38)$$

To derive an expression for S, we consider the constituent parts of Equation (38) separately. First:

$$\begin{aligned}
&\mathrm{E}\,[\,Q\,U_\Gamma(W-\Pi L)\,] \\
&= \sum_{i=1}^{n} \mathrm{E}[\,Q\,U_\Gamma(W-\pi_i L) \mid \mu=\mu_i\,]\,\mathrm{P}[\mu=\mu_i], && (39)\\
&= \sum_{i=1}^{n} \{\mathrm{E}[\,U_\Gamma(W-\pi_i L) \mid U_\Gamma(W-\pi_i L) > u_{c_i}\,,\, \mu=\mu_i\,] && (40)\\
&\qquad\qquad \times \mathrm{P}\,[U_\Gamma(W-\pi_i L) > u_{c_i} \mid \mu=\mu_i\,]\; p_i, \} \\
&= \sum_{i=1}^{n} U_i^*(W-\pi_i L)\,d_i(\pi_i)\,p_i, \quad \text{using Equation (5)}, && (41)
\end{aligned}$$

where $u_{c_i} = (1-\mu_i)U(W) + \mu_i U(W-L)$ (as defined in Equation (4)) and $U_i^*(W-\pi_i L) = \mathrm{E}[\,U_\Gamma(W-\pi_i L) \mid U_\Gamma(W-\pi_i L) > u_{c_i}\,,\, \mu=\mu_i\,]$ represents the expected utility of individuals purchasing insurance in risk-group i.

The second part of Equation (38) can be written as:

$$\begin{aligned}
&\mathrm{E}\,[\,(1-Q)\,[(1-X)\,U(W) + X\,U(W-L)]\,] \\
&= \sum_{i=1}^{n} \mathrm{E}[\,(1-Q)\,[(1-X)\,U(W) + X\,U(W-L)] \mid \mu=\mu_i\,]\,\mathrm{P}[\mu=\mu_i], && (42)\\
&= \sum_{i=1}^{n} [(1-d_i(\pi_i))\,\{(1-\mu_i)U(W) + \mu_i\,U(W-L)\}]\,p_i && (43)
\end{aligned}$$

Combining Equations (41) and (43), we get the following expression for social welfare:

$$S = \sum_{i=1}^{n} \Bigg[\underbrace{d_i(\pi_i)\, U_i^*(W - \pi_i\, L)}_{\text{Insured population}} \tag{44}$$

$$+ \underbrace{(1 - d_i(\pi_i))\, \{(1-\mu_i)U(W) + \mu_i\, U(W-L)\}}_{\text{Uninsured population}} \Bigg] p_i,$$

$$= \underbrace{\sum_{i=1}^{n} [(1-\mu_i)U(W) + \mu_i\, U(W-L)]\; p_i}_{\text{Constant as a function of } \pi_i} \tag{45}$$

$$+ \underbrace{\left(\sum_{i=1}^{n} d_i(\pi_i)\mu_i p_i \right) \times [U(W) - U(W-L)]}_{\text{Loss coverage} \times \text{Positive multiplier}}$$

$$- \underbrace{\sum_{i=1}^{n} d_i(\pi_i)\, [U(W) - U_i^*(W - \pi_i L)]\; p_i}_{\text{Adjustment factor to account for premiums}} .$$

$$= \text{Constant} + \textbf{Loss Coverage} \times \text{Positive multiplier} \tag{46}$$

$$- \text{Premium adjustment factor.}$$

Note that Equation (46) does not depend on any particular choice of family of utility functions.

A regulator or a policymaker aiming to maximise social welfare will be interested in choosing a risk-classification regime $\underline{\pi}$ which maximises S. However, social welfare depends on unobservable utility functions, which makes it difficult to implement. On the other hand, loss coverage depends solely on observable quantities and Equation (46) shows that social welfare and loss coverage are directly related. So it will be useful if it can be shown that both measures, social welfare and loss coverage, agree on the choice of risk-classification regime under certain assumptions.

6.2 Iso-elastic demand: Social welfare and loss coverage

Using the convenient standardisation of $U(W) = 1$ and $U(W - L) = 0$ as defined in Equations (7) and (8), along with the assumption that $W = L = 1$, and noting that social welfare S is a function of the risk-classification regime $\underline{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$, Equation (38) becomes:

$$S(\underline{\pi}) = E\left[\, Q\, U_\Gamma(1 - \Pi) + (1 - Q)(1 - X)\,\right], \tag{47}$$

$$= E\left[\, Q\, \{U_\Gamma(1 - \Pi) - (1 - X)\,\}\,\right] + K, \tag{48}$$

where $K = E[1 - X]$ is a constant as it does not depend on $\underline{\pi}$.

Using the utility function $U(w) = w^\gamma$ which corresponds to iso-elastic demand, we have:

$$
\begin{aligned}
S(\underline{\pi}) &= E\left[\, Q\, \left\{ (1-\Pi)^{\Gamma} - (1-X)\, \right\} \right] + K, && (49)\\
&\approx E\left[\, Q\, (1 - \Gamma\, \Pi - 1 + X\,)\,\right] + K, \quad \text{assuming small premiums}, && (50)\\
&= E\left[\, Q(X - \Gamma\, \Pi)\,\right] + K, && (51)\\
&= E\left[\, QX\,\right] - E\left[\, Q\, \Gamma\, \Pi\,\right] + K, && (52)\\
&= LC(\underline{\pi}) - PA(\underline{\pi}) + K, && (53)
\end{aligned}
$$

where $LC(\underline{\pi}) = E\,[\,QX\,]$ is loss coverage and $PA(\underline{\pi}) = E\,[\,Q\,\Gamma\,\Pi\,]$ is the premium adjustment factor under the risk-classification regime $\underline{\pi}$. We have already analysed $LC(\underline{\pi})$ in Equation (33), so we focus on $PA(\underline{\pi})$ here.

First, recall that Q is an indicator random variable which takes the value of 1 if insurance is purchased; 0 otherwise. And from Equation (16), given a risk-group i, insurance is purchased when $\Gamma_i < \frac{\mu_i}{\pi_i}$, where the random variable $\Gamma_i = [\,\Gamma \mid \mu = \mu_i\,]$. Hence:

$$
[\,Q \;\mid\; \mu = \mu_i\,] = I\left[\Gamma_i < \frac{\mu_i}{\pi_i}\right]. \qquad (54)
$$

So:

$$PA(\underline{\pi}) = \sum_{i=1}^{n} E\,[\,Q\,\Gamma\,\Pi \mid \mu = \mu_i\,]\; P[\mu = \mu_i], \tag{55}$$
$$= \sum_{i=1}^{n} E\left[I\left[\Gamma_i \leq \frac{\mu_i}{\pi_i}\right]\,\Gamma_i\,\pi_i\right]\,p_i, \tag{56}$$
$$= \sum_{i=1}^{n} E\left[\Gamma_i\,I\left[\Gamma_i \leq \frac{\mu_i}{\pi_i}\right]\right]\,\pi_i\,p_i. \tag{57}$$

Using the cumulative distribution function of Γ_i, as given in Equation (17):

$$P\,[\Gamma_i \leq \gamma] = \begin{cases} 0 & \text{if } \gamma < 0 \\ \tau_i\,\gamma^{\lambda} & \text{if } 0 \leq \gamma \leq (1/\tau_i)^{1/\lambda} \\ 1 & \text{if } \gamma > (1/\tau_i)^{1/\lambda}, \end{cases} \tag{58}$$

Equation (57) becomes:

$$PA(\underline{\pi}) = \sum_{i=1}^{n}\left[\int_0^{\frac{\mu_i}{\pi_i}} \gamma\tau_i\lambda\gamma^{\lambda-1}d\gamma\right]\pi_i p_i, \tag{59}$$
$$= \frac{\lambda}{(\lambda+1)}\sum_{i=1}^{n}\left(\frac{\mu_i}{\pi_i}\right)^{\lambda+1}\tau_i\pi_i p_i, \tag{60}$$
$$= \frac{\lambda}{(\lambda+1)}\sum_{i=1}^{n}\frac{\mu_i^{\lambda+1}}{\pi_i^{\lambda}}\tau_i p_i, \tag{61}$$
$$= \frac{\lambda}{(\lambda+1)}LC(\underline{\pi}), \quad \text{by Equation (33).} \tag{62}$$

Hence social welfare in Equation (53) becomes:

$$S(\underline{\pi}) = \frac{1}{\lambda + 1} LC(\underline{\pi}) + K. \tag{63}$$

The right-hand side Equation (63) can be interpreted as follows. The second term $K = E[1 - X]$ corresponds to expected utility in the absence of the institution of insurance (recall that we have standardised $U(W) = 1$, $U(W - L) = 0$, and X is the loss for an individual drawn at random from the population). The first term represents an increase in expected utility, attributable to the institution of insurance; this allows for the expectations of both utility of benefits received, and disutility of premiums paid. If λ is small (corresponding to inelastic demand and high risk aversion), the premiums paid are relatively unimportant, so the increase in expected utility is a large fraction of the loss coverage [8]. If λ is large (corresponding to elastic demand and low risk aversion), the premiums paid are important, so the increase in expected utility is only a small fraction of the loss coverage.

The form of Equation (63) suggests the following proposition.

Proposition 2. *Suppose demand elasticity is a positive constant and we have two risk classification schemes $\underline{\pi}_1$ and $\underline{\pi}_2$, which give social welfare $S(\underline{\pi}_1)$ and*

[8] The fraction $1/(\lambda + 1)$ can also be viewed as a fraction of the loss coverage $LC(\underline{\pi}) = E[QX]$ which 'counts' as an offset against the uninsured losses X which appear in $K = E[1 - X]$, where the offset is in on a welfare scale and includes allowance for both benefits and premiums.

$S(\underline{\pi}_2)$, and loss coverage $LC(\underline{\pi}_1)$ and $LC(\underline{\pi}_2)$. Then

$$S(\underline{\pi}_1) \geq S(\underline{\pi}_2) \Leftrightarrow LC(\underline{\pi}_1) \geq LC(\underline{\pi}_2) \tag{64}$$

In other words: for iso-elastic insurance demand, ranking risk classification schemes by loss coverage always gives the same ordering as ranking by social welfare.

The proof of the propositon follows directly from the form of Equation (63), and noting that for the logical biconditional statement in the proposition, the contrapositive (i.e with both inequalities reversed) also holds.

Proposition 2 holds for *any* pair of risk classification schemes which satisfy the equilibrium condition in Equation (28). This includes schemes where premiums are partly (but not fully) risk-differentiated, as well as the polar cases of pooling and actuarially fair premiums. Where the comparison is between the polar cases, combining Proposition 2 with Proposition 1 shows that for iso-elastic demand, pooling gives higher social welfare than actuarially fair premiums whenever demand elasticity is less than one, and *vice versa*.

The potential usefulness of Proposition 2 arises from the fact that loss coverage is observable, but social welfare is unobservable. So a policymaker or regulator can implement a risk classification scheme which gives higher (observable) loss coverage, with the comfort of knowledge that this also gives higher (unobservable) social welfare.

7 Conclusions

We have proposed loss coverage as an intuitively appealing metric for evaluation of different insurance risk classification schemes. Loss coverage is defined as the expected population losses compensated by insurance at market equilibrium.

Bans on insurance risk classification create asymmetries in (the use of) information, typically leading to adverse selection. Adverse selection is associated with a fall in the number of insured individuals compared with that obtained under full risk classification. This reduction in coverage is usually seen as inefficient. However, adverse selection is also associated with a shift in coverage towards higher risks. If this shift is large enough, it can more than outweigh the fall in numbers insured, so that loss coverage is increased. Since this implies that more risk is voluntarily traded and more losses are compensated, it is a counter-argument to the perception of reduced efficiency.

For coverage to shift towards higher risks when risk classification is banned, it must be the case that not all individuals choose to buy insurance at any given premium. This is an observable reality in many insurance markets. We have shown that it can be explained by heterogeneous utility functions, which are unobservable by the insurer. In our model, individuals make decisions completely deterministically on the basis of certainty-equivalent utility calculations, but the insurer observes apparently stochastic decision-making, resulting in a proportional insurance demand function.

We have also shown that loss coverage can be reconciled with (although it is not the same as) an 'equal weights' utilitarian social welfare, in the spirit of Hoy [2006] or Dionne and Rothschild [2014]. Specifically, if insurance demand is iso-elastic, ranking risk classification schemes by loss coverage always gives the same ordering as ranking by social welfare. Notably, however, the calculation of social welfare requires utility functions to be observable, while the calculation of loss coverage does not.

This work could be extended in both empirical and theoretical directions. Empirically, we could investigate how reasonable the iso-elastic model is as an approximation for insurance demand in particular markets. Theoretically, it may be possible generalise our main results for a wider class of insurance demand functions. However, both these extensions are left for future research.

Appendices

A Loss Coverage Ratio

The argument given here follows Hao *et al.* (2016). The loss coverage ratio for the case of equal demand elasticity is given in Equation (35) and can be expressed as follows:

$$C = \frac{1}{\pi_0^\lambda} \frac{\sum_{i=1}^n \alpha_i \mu_i^{\lambda+1}}{\sum_{i=1}^n \alpha_i \mu_i}, \quad \text{where} \quad \pi_0 = \frac{\sum_{i=1}^n \alpha_i \mu_i^{\lambda+1}}{\sum_{i=1}^n \alpha_i \mu_i^\lambda}; \tag{65}$$

$$= \left[\sum_{i=1}^n w_i \mu_i^{\lambda-1}\right]^\lambda \left[\sum_{i=1}^n w_i \mu_i^\lambda\right]^{1-\lambda}, \quad \text{where} \quad w_i = \frac{\alpha_i \mu_i}{\sum_{j=1}^n \alpha_j \mu_j}; \tag{66}$$

$$= \left(E_w\left[\mu^{\lambda-1}\right]\right)^\lambda \left(E_w\left[\mu^\lambda\right]\right)^{1-\lambda}, \tag{67}$$

where E_w denotes expectation in this context and the random variable μ takes values $\mu_1, \mu_2, \ldots, \mu_n$ with probabilities $w_1, w_2, \ldots, w_n$ respectively.

Result A.1. *For $\lambda > 0$,*

$$\lambda \lesseqqgtr 1 \Leftrightarrow C \gtreqqless 1. \tag{68}$$

Proof. **Case $\lambda = 1$:** It follows directly from Equation (67) that $C(1) = 1$.

Case $0 < \lambda < 1$: Holder's inequality states that, if $1 < p, q < \infty$ where $1/p + 1/q = 1$, for positive random variables X, Y with $E[X^p], E[Y^q] < \infty$, $\left(E\left[X^p\right]\right)^{1/p} \left(E\left[Y^q\right]\right)^{1/q} \geq E[XY]$.

Setting $1/p = \lambda$, $1/q = 1 - \lambda$, $X = \mu^{\lambda(\lambda-1)}$ and $Y = 1/X$, applying Holder's inequality to Equation (67) gives,

$$C = \left(E_w\left[X^{1/\lambda}\right]\right)^\lambda \left(E_w\left[Y^{1/(1-\lambda)}\right]\right)^{1-\lambda} \geq E_w[XY] = 1. \tag{69}$$

Case $\lambda > 1$: Lyapunov's inequality states that, for positive random variable μ and $0 < s < t$, $\left(E[\mu^t]\right)^{1/t} \geq \left(E[\mu^s]\right)^{1/s}$.

So Equation 67 gives:

$$C = \frac{\left(E_w\left[\mu^{\lambda-1}\right]\right)^{\lambda}}{\left(E_w\left[\mu^{\lambda}\right]\right)^{\lambda-1}} = \left[\frac{\left(E_w\left[\mu^{\lambda-1}\right]\right)^{1/(\lambda-1)}}{\left(E_w\left[\mu^{\lambda}\right]\right)^{1/\lambda}}\right]^{\lambda(\lambda-1)} \leq 1, \qquad (70)$$

as $\left(E_w\left[\mu^{\lambda-1}\right]\right)^{1/(\lambda-1)} \leq \left(E_w\left[\mu^{\lambda}\right]\right)^{1/\lambda}$ for $\lambda > 1$ by Lyapunov's inequality.

□

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