Valuation of no-negative-equity guarantees with a lower reflecting barrier

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Abstract

If the general level of house prices falls a long way, policymakers may introduce new policies which seek to support prices. This paper considers the effect of such interventions on the valuation of no-negative-equity guarantees (NNEG) in equity release mortgages. I discuss past examples of interventions, and policymaker statements which suggest the prospect of future interventions. I model interventions by a reflecting barrier expressed as a fraction of the current level of house prices. Reflection at the barrier is instantaneous, so the no-arbitrage property is preserved, and hence risk-neutral valuation of NNEG is possible. The reflecting barrier can alternatively be justified as a representation of the different economic nature of the underlying housing (and particularly freehold land) assets in NNEG valuations, compared with the underlying equity assets in many other option valuations.

Keywords: No-negative-equity guarantee; NNEG; Equity release; Lifetime mortgage; Put option; Reflecting barrier

1. Introduction

1.1 Motivation

In 2018 I wrote a blog post on valuation of no-negative-equity guarantees (NNEG) in equity release mortgages, which included the following passage:

> "[A] deep and prolonged fall in house prices, with the attendant collapse in mortgage lending, widespread repossessions and distress in the electorate, seems overwhelmingly likely to induce a policymaker response. This is illustrated by the policymaker response to the modest fall in house prices following the 2008 financial crisis: policies such as purchasing first gilts and then corporate bonds (and even equities in Japan); the term funding scheme to revive mortgage lending; and policies such as help-to-buy and associated schemes providing blatantly direct support for house prices. And all this was in response to a quite modest and

short fall in prices! In a deeper or more prolonged slump, there are many more steps which policymakers can (and I believe will) take. In a country with its own currency, the government can (and I believe will) ultimately print money and buy houses. The activities and statements of central bankers worldwide in relation to asset purchases in recent years provide further general support for this notion of a policy response to deep and prolonged falls in asset (and particularly house) prices.

This is not the same as saying that I think house prices will always go up, or that housing is a better investment than shares, or that you can never lose by buying a house. I do not believe any of that. I just believe that overwhelmingly likely policymaker responses now provide a reflecting barrier under house prices, which makes geometric Brownian motion an unreasonable assumption for house prices in the lower tail. The level and firmness of that barrier is matter for reasonable debate, but it seems unrealistic to pretend – as the prescribed NNEG formula does – that it does not exist at all." (Thomas, 2018)

Since then several papers and notes on equity release mortgages and NNEG in the specific United Kingdom and Ireland context have been published (e.g. Tunaru and Quaye 2019, Jeffery and Smith 2019, Buckner and Dowd 2019, Dowd et al 2019, Turnbull 2019a, b, c). These papers discuss many aspects of equity release mortgages and NNEG valuation. But as far as I am aware, no published work has engaged directly with the ideas in the passage above. There are several possible reasons for this. First, others may regard the prospect of future policymaker intervention in the housing market as less likely or less material than I do. Second, they may agree that the idea of intervention is plausible, but take the view that no credit should be taken for it in valuation. Third, they may think the idea is mathematically intractable; but mathematically intractable ideas are not necessarily untrue (and sometimes very valuable in an investment context). In my view, it remains of interest to consider the potential effect of policymaker interventions on house prices and hence valuation of NNEG.

This paper models such interventions by means of a lower reflecting barrier under house prices, expressed as a fraction of the level of prices today. Hertrich (2015) and Hertrich & Zimmermann (2017) give a formula for risk-neutral valuation of a put option in the presence of such a barrier. They consider a currency option, where the barrier represents a floor under the exchange rate which is maintained by a policy of currency intervention by the central bank. Such floors can never be absolute; there is always a risk that the central bank might exhaust its reserves, or change its policy with regard to defending the floor. But there seems little reason to think that a model which simply ignores the policy floor will give a sensible valuation of a currency put. Similarly, interventions in the housing market cannot place completely firm barrier under prices. But there seems little reason to think that just ignoring the likely political response to large falls in house prices will give a sensible valuation of NNEG.

This paper adapts Hertrich & Zimmermann's approach to the NNEG context. The key point is that assuming a reflecting barrier even as much as 50% below today's price level can substantially reduce the value of NNEG. Stated differently: much of the Black-Scholes value of NNEG may be attributable to extreme scenarios where house prices do not merely stagnate or fall, but collapse by 50% or more over the term of the NNEG. If we expect policymakers to intervene in this type of scenario with attempts to support house prices, valuations of NNEG made without consideration of policymaker actions may be overstated.

1.2 Institutional and regulatory background

An equity release mortgage is a product whereby an individual borrower typically aged 60 years or more borrows 25-30% of the valuation of their house from an institutional lender, typically an insurance company. The alternative names 'lifetime mortgage' (in the United Kingdom) or 'reverse mortgage' (in the United States and Australia) are sometimes used. In the United Kingdom, the most common product design involves a fixed rate of interest charged to the home owner, but not actually paid. The loan is secured by a first charge on the house. On the borrower's death or any earlier sale of the house (e.g. after the borrower moves into a care home), the loan plus accumulated interest is repayable (typically from the sale proceeds of the house).

In early designs of equity release mortgage in the 1980s, if the loan plus accumulated interest exceeded the sale proceeds of the house, the lender could claim the excess from the borrower's residual estate. This feature came to be perceived as problematic for both borrower and lender. The product feature of a no-negative-equity guarantee was therefore introduced, and is now a mandatory requirement for members of United Kingdom trade association, the Equity Release Council. The NNEG guarantees that the amount repayable by the borrower (or estate after their death) will not exceed the sale proceeds of the house.

The NNEG allows the borrower to repay the lower of two amounts (the rolled-up loan, or the sale proceeds of the house). So the lender's offer of the guarantee is akin to writing a put option on the house price over the term to the borrower's date of death, with a strike price defined as the rolled-up loan at the date of death. Although valuation methods for short-term put options which can be continuously hedged by short sales of the underlying are well known and widely accepted, the NNEG relates to much longer terms, and the underlying asset (the house) cannot readily be sold short. The application of standard option pricing theory to NNEG valuation is therefore controversial. Starting in 2016, the UK regulator, the Prudential Regulation Authority PRA), has issued a number of documents touching on NNEG valuation.[1]

The controversy arises principally because the inability to sell short implies that the continuous hedging argument which underlies most justifications of risk-neutral option pricing is not realistic for NNEG. There are some other possible justifications, as outlined in e.g. Andreasen et al (1998) or

[1] These include PRA Discussion Paper 1/16; Consultation Papers 48/16, 13/18, and 7/19; Policy Statement 19/19; and Supervisory Statement 3/17.

Wilmott (2009, chapter 7). But all seem to rely to some degree on assumptions which are not realistic for NNEG. Perhaps the most engaging alternative is that if the underlying asset returns are lognormal, the Black-Scholes formula (or variants thereof) can be inferred without continuous hedging, by assuming a requirement for no arbitrage between prices of puts, calls and forward contracts with common strike prices (Derman & Taleb, 2005). But even this is not compelling for NNEG, because the underlying asset returns are probably not lognormal, and traded puts, calls and forwards on house prices simply don't exist.

Despite these difficulties, a regulator may nevertheless choose to mandate that options should be valued on a risk-neutral basis. This is what the PRA has done for NNEG, at least for the purposes of its Effective Value Test. The Effective Value Test places a limit on the benefit which insurers can obtain by restructuring equity release mortgages to make them eligible assets for the Matching Adjustment[2] under Solvency II. The Effective Value Test requires that

$$\begin{pmatrix} \text{value of restructured mortgages} \\ \text{on asset side of balance sheet} \\ + \\ \text{value of Matching Adjustment} \\ \text{on liability side of balance sheet} \end{pmatrix} < \begin{pmatrix} \text{economic value of} \\ \text{unrestrucured mortgages} \end{pmatrix}$$

where the economic value on the right-hand side must include an allowance for NNEG using a prescribed Black-Scholes type formula (i.e. on a risk-neutral basis) and prescribed minimum parameters (PRA Supervisory Statement 3/17, para 3.20).

In the light of this requirement, this paper starts from the usual risk-neutral paradigm for option pricing, and makes the minimum changes necessary to accommodate a reflecting barrier under prices. I adopt this approach for reasons of consistency and comparability with the regulatory framework. I take the apparent regulatory preference for the principle of risk-neutral valuation as given, and do not further discuss the arguments for or against the principle in the context of long-term unhedgeable options.

This paper focuses on the option valuation aspect of NNEG, in the context of the regulatory background just described. As far as possible, all surrounding detail which is not specific to option valuation is left aside. In practice a NNEG always has an unknown future term (the time until the borrower's death or earlier repayment of the loan), and so is evaluated by summing the product of option valuation and exit probability over all possible terms. But I ignore this aspect, and instead

[2] The Matching Adjustment is an increase in the permitted discount rate for liabilities in the Solvency II balance sheet, which is available where the liabilities are backed by long term illiquid assets with fixed cashflows. Equity release mortgages are contingent on borrower mortality and so do not provide fixed cashflows. But by restructuring them into several tranches, the senior tranches can be assigned fixed cashflows, and so achieve eligibility for the Matching Adjustment.

focus on option valuation over an assumed certain term. I give no consideration to valuation of the risk-free loan component of equity release, or the calibration of parameters other than volatility and the reflecting barrier. I also do not discuss the restructuring of equity release mortgages into senior and junior tranches as alluded to above.

1.3 Literature review

The NNEG product feature became prevalent in equity release products only in the early 2000s, so all literature is relatively recent. Hosty et al (2008) is an early United Kingdom practitioner perspective, which covers much institutional detail and suggests two alternatives for valuation: risk-neutral valuations using standard Black-Scholes type formulae, and 'real-world' valuations. The difference between these is as follows. One of the principal inputs to Black-Scholes type formulae is the forward price of the house, i.e. the price agreed now to take possession of the house at the end of the term, with payment at the end of the term. The 'real-world' valuation uses the same formula, but replaces the forward price with a projected house price at the maturity date, based on an assumption for house price growth. This typically leads to much lower valuations than risk-neutral valuation, and has been a common approach in practice (at least until recent regulatory guidance).

The academic literature focuses mainly on risk-neutral valuation of NNEG. Technically speaking, the main difficulty with risk-neutral valuation is the lack of market prices for forwards and options on house prices; this implies that the market is incomplete, and hence there is no unique risk-neutral measure. Much of the literature can be grouped according to the approach used to select a risk-neutral measure, as follows.

- *Esscher transform:* Li et al (2010) note that lognormal models are unrealistic for house prices, and instead fit an ARMA-EGARCH model to the Nationwide House Price Index, with a conditional Esscher transform to obtain a risk-neutral measure. Other papers using the Esscher transform approach include Chen et al (2010). Lee et al (2012, 2018) and Tunaru & Quaye (2019).
- *Stochastic discount factors:* Alai et al (2014) use a vector autoregressive model of house prices, rental yields, nominal GDP, and short and long term interest rates, and then price the NNEG by using stochastic discount factors derived from this model (see also the overlapping group of authors Shao et al (2015)).
- *Canonical valuation (maximum entropy) principle:* Kogure et al (2014) and Kim & Li (2017) derive a risk-neutral measure by a principle known as canonical valuation. Intuitively, this principle says that when choosing one of the many equivalent martingale measures which exist in an incomplete market, we should choose the measure which is 'closest' (in a certain technical sense) to the real-world measure.

In contrast to this literature, the present paper assumes a geometric Brownian motion for house prices, which makes the risk-neutralisation step straightforward. The barrier concept is then superimposed on this, and does not require any additional risk-neutralisation (for reasons explained later in the paper).

In the United Kingdom, the PRA's consultations and guidance from 2016 onwards prompted renewed interest in NNEG valuation. In a review paper jointly commissioned by the Institute and Faculty of Actuaries and the Association of British Insurers, Tunaru and Quaye (2019) advocate a similar ARMA-EGARCH approach to Li et al (2010), but with a slightly different lag specification for the ARMA part of the model. Jeffery and Smith (2019) review NNEG valuation in the context of both Ireland and United Kingdom experience, and question the common views that equity release is a good hedge for annuity liabilities, and that the availability of equity release is indisputably in the public interest ("we believe the case is finely balanced"). Buckner and Dowd (2019) make a trenchant argument for risk-neutral valuation, and in particular for the valuation formula (although not the minimum calibration) prescribed by the PRA, rather than more complex approaches.

As already noted, the main sources for put option pricing in the presence of a reflecting barrier are Hertrich (2015) and Hertrich & Zimmerman (2017). In the actuarial literature, Gerber and Pafumi (2000) consider a closely related problem. They value a dynamic guarantee on an investment fund, where an infinitesimal amount of money is added instantaneously to the fund every time it falls to the guarantee level, so as to prevent it ever falling below the guarantee. This is analogous to the intervention which substantiates the barrier in the present paper. Their problem is simpler because they have only a barrier, whereas we have an option strike *and* a lower barrier, so they obtain a simpler (but clearly analogous) valuation formula. Imai and Boyle (2001) obtain the same formula by a different argument, and consider extensions for discrete monitoring of the barrier, constant elasticity of variance processes, and American-style guarantees.

The rest of this paper is organised as follows. Section 2 outlines past examples of policymaker intervention in housing markets, and policymakers' public statements which suggest that future intervention after a large fall in prices may be plausible. Section 3 discusses possible alternative rationales (besides policy intervention) for assuming a reflecting barrier. Section 4 gives some initial intuition for how a reflecting barrier might affect NNEG values. Section 5 gives a risk-neutral valuation formula for NNEG with a lower reflecting barrier. Section 6 presents examples and sensitivity tests. Section 7 considers what level of barrier to use. Section 8 discusses applications, and touches on other approaches to allowing for policymaker actions, notably Economic Scenario Generator models. Section 9 concludes.

2. Policy interventions to support house prices: past, present and future

This section outlines the range of policy interventions in the housing market in the United Kingdom that have been observed over the past few decades, and the prospects for similar or more extensive interventions in future.

2.1. Three types of intervention

The housing market has been the target of policy intervention for at least as long as any available data series on the general level of prices. At a very general level, owner occupation has been long perceived to be a natural aspiration of many voters, and moreover one for which the State should provide support. It does so through institutional arrangements, fiscal policies, and monetary policies. These categories are not wholly distinct – the implementation of monetary and fiscal policies necessarily involves institutional arrangements – but they nevertheless provide a convenient taxonomy.

2.1.1 Institutional arrangements

Debt finance is widely available for purchase of owner-occupied housing on preferential terms compared with other assets. For a person with a clean credit record, it is generally straightforward to borrow between three and five times one's annual income to buy a single undiversified house. It is more difficult and more expensive – for many people, impossible – to borrow even half one's annual income to buy a diversified portfolio of other assets. Debt finance for rented houses has also become easier to obtain in the past two decades. The Housing Act 1988 removed security of tenure, giving landlords greater freedom to evict tenants; this made rented houses better security for loans, and so gave rise to the phenomenon of 'buy to let' (a phrase which first appears in the Lexis database of British national newspapers only in the year 1995).

The greater availability of debt for housing compared with other assets is facilitated by many institutional and regulatory choices. Turner (2016, Chapter 4) discusses the many ways in which banking regulation incentivises banks to prefer lending for purchase of existing houses rather than other purposes, in particular the very low regulatory risk weights for loans secured against housing. As a consequence, residential mortgages are more than 50% of bank lending in the UK, whilst lending to finance non-property capital investment is probably less than 15% (Turner 2016, p62).

2.1.2 Fiscal policies

Fiscal policy can be used to support house prices in many ways. In the past this included tax relief on mortgage interest; the gradual phasing out of this between 1994 and 2000 was a rare example of the withdrawal of a form of policy support for house prices. Capital gains on owner-occupied housing enjoy an unlimited tax exemption. There is no land value tax. Property tax (council tax) is set at very low levels, and is regressive relative to its base (i.e. the more a house is worth, the smaller the proportion of its value that is paid in council tax). The housing services which owner occupiers consume are not taxed, that is imputed rents are not taxed. To the reader familiar only with the contemporary UK housing market, some of these points may seem obscure and theoretical, but policy choices are different elsewhere. Land value taxes and property taxes are higher in many jurisdictions. Imputed rents were formerly taxed in the UK under 'Schedule A' (albeit at a low value, and abolished in 1963), and are taxed today in Switzerland, where the proportion of the housing stock that is owner-occupied has been stable around 25% throughout past half-century (Jordà et al (2016), p122.)

The combination of tax-free capital gains and tax-free imputed rents encourages voters to allocate substantially all of their asset portfolios to a single house rather than to a more diversified portfolio of assets. The preferential availability of debt finance for house purchase encourages leveraging of this allocation. These phenomena are so familiar and long-standing that they are often regarded as natural and inevitable; but in reality they are policy choices, which I believe policymakers will attempt to justify and support with novel interventions if they start to go wrong. Indeed, we have already seen this after the modest fall in houses prices starting in 2008.

From 2008, a succession of schemes under governments of all complexions have seen the State increasingly take a direct role as a provider of low-cost finance for house purchase. The Help to Buy scheme, introduced in 2013 and available to all purchasers of newly-built houses priced under £600,000 in England (£300,000 in Wales), is the most familiar of these schemes today. It provides an interest-free equity loan for five years at a 20% loan-to-value (LTV) ratio, or 40% in Greater London. It was preceded by smaller schemes targeted at first time buyers, including the now largely forgotten Home Buy Direct, introduced by a Labour government, which offered 30% LTV interest-free equity loan for five years from September 2008; and the First Buy scheme, which offered 20% LTV interest-free equity loans for five years from 2011. Contrary to common belief, the Help to Buy scheme is not restricted to purchasers who are first-time buyers (and around 20% are not). The effect of all these schemes, at least in part, has been to support house prices.[3]

[3] Carozzi et al (2019) evaluate the effects of Help to Buy by exploiting the differences in its design in adjacent geographical areas. They find that along the border of the Greater London area, the main effect was to increase prices of new builds just inside the area by about 5% (compared with new builds just outside the area), with no effect on construction volumes. On the English/Welsh border, where land supply is less constrained, there was some increase in construction volumes, but no effect on prices.

Some critics regard Help to Buy as poorly targeted and inefficient (e.g. National Audit Office, 2014, 2019). But more than a decade after the first similar scheme was introduced, there is little sign of Help to Buy being curtailed. It was originally intended as a short term scheme, but on current plans it will last until at least 2023, and consume over eight times its original budget. A cross-party committee of MPs has recently demanded that 'alternative schemes' must be introduced when Help to Buy ends in 2023 (Public Accounts Committee, 2019). This illustrates the point that interventions to support house prices tend to ratchet upwards; there is an electoral imperative to introduce them when prices fall, and a reluctance to curtail them when prices recover.

2.1.3 Monetary policies

The fiscal policies just described were implemented in the context of a relatively shallow and short dip in house prices (21% as measured by the Halifax All UK index from Q3 2007 to Q1 2009). In a deep and prolonged slump which leads to widespread negative equity, repossessions, and distress amongst the electorate, similar but more dramatic fiscal policy changes could be made. There are also many possible monetary policy actions, which are of two main types: bank-focused and market-focused.

Bank-focused measures primarily relax operating constraints on banks, on the rationale that this will stimulate bank lending and hence activity in the real economy. Any effect on asset prices is secondary and indirect, but can nevertheless be substantial. Bank-focused measures include official interest rate cuts, open market operations (i.e. purchase of short term government bonds from banks, increasing the monetary base), relaxation of collateral requirements (i.e. the central bank accepts illiquid or low quality collateral when making loans to banks), and the granting of non-recourse loans to banks.

Market-focused measures primarily involve purchasing long duration assets, and so lowering long term interest rates and raising share prices. The justification is that this leads to positive confidence and wealth effects which stimulate activity in the real economy. Market-focused measures are often described as 'quantitative easing' (the focus is on increasing the *quantity* of credit in the economy, rather than reducing the *price* of that credit, i.e. the interest rate).[4] The notion that policy directly seeks to raise share prices may seem like a cynical allegation, but it has been explicitly embraced by policymakers at the highest level. Here is Ben Bernanke, then chairman of the Federal Reserve, explaining quantitative easing in an article in 2010:

[4] My parsing of the needlessly opaque phrase 'quantitative easing' reflects the original etymology as follows. In 1995 Richard Werner, then chief economist at Jardine Fleming in Japan, wrote an article for the Nikkei newspaper proposing that the Bank of Japan should pursue policies to increase the quantity of credit creation by banks, rather than reducing the price of money or increasing the monetary base. One such policy was to purchase non-performing loans from banks. Because the expression 'credit creation' was difficult to understand in Japanese, Werner combined an adjective for 'quantitative' with a standard term for 'monetary stimulation'. 'Quantitative easing' was a literal translation into English of the two Japanese words. The phrase was subsequently adopted by other commentators to cover official purchases of long duration assets in general, rather than purchases of non-performing loans in particular. https://www.res.org.uk/resources-page/july-2013-newsletter-quantitative-easing-and-the-quantity-theory-of-credit.html, accessed 16 November 2019.

> "Easier financial conditions will promote economic growth. For example lower mortgage rates will make housing more affordable and allow more homeowners to refinance. Lower corporate bond rates will encourage investment. And higher stock prices will boost consumer wealth and help increase confidence, which can also spur spending." [5]

Quantitative easing was largely unknown before 2008, and originally presented as a short-term response to a banking crisis. But more than a decade later, policymakers increasingly seem to view it as a routine policy tool. A number of current and former central bankers have noted that with official interest rates already close to zero, the policy response to future recessions may need to include purchases of a wider range of assets than in the past. This is old news in Japan: the Bank of Japan has been buying equities via Exchange Traded Funds on a large scale since 2013 (with smaller purchases starting in 2010), and has now accumulated around 5% of the Japanese equity market. Here are four more examples of policymakers contemplating the purchase of a wider range of assets:

- When the Bank of England's Asset Purchase Facility was extended in August 2016 to include the purchase of up to £10bn of corporate bonds, the minutes of the Monetary Policy Committee explicitly noted that the "The MPC could act further…by expanding the scale or variety of the asset purchases".[6]

- In a speech on 26 August 2016, Janet Yellen, then Chair of the Federal Reserve, said that "future policymakers may wish to explore the possibility of purchasing a broader range of assets."[7] When asked to elaborate in an interview some weeks later, she replied: "The idea of expanding into areas like equities might be a good thing to think about…it could be useful to be able to intervene directly in assets where the prices have a more direct link to spending decisions". (The Federal Reserve is not currently authorised by Congress to buy equities.)[8]

- In a speech in 2016, former US Treasury Secretary Lawrence Summers suggested that central banks should consider purchasing "a wider range of assets on a sustained and continuing basis".[9]

- Sir Charles Bean, the former Bank of England governor and chief economist, in a lecture (subsequently printed as an article) in 2018, said: "Could the central bank ever run out of assets to

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[5] Bernanke, B. (2010) 'What the Fed did and why: supporting the recovery and sustaining price stability', *Washington Post*, 4 November 2010.

[6] Bank of England (2017) Minutes of the Monetary Policy Committee meeting ending on 3 August 2016. https://www.bankofengland.co.uk/monetary-policy-summary-and-minutes/monetary-policy-summary-and-minutes, accesses 12 October 2019.

[7] 'The Federal Reserve's Monetary Policy Toolkit: Past, Present, and Future' Speech dated 26 August 2016. https://www.federalreserve.gov/newsevents/speech/yellen20160826a.htm, accessed 12 October 2019.

[8] 'A Conversation with the Federal Reserve Chair Janet Yellen'. Video interview dated 28 September 2016 (starting at 9:00). https://www.kansascityfed.org/community/events/minority-bankers-forum, accessed 12 October 2019.

[9] Summers floats idea of sustained government stock purchases', Bloomberg, 30 September 2016. https://www.bloomberg.com/news/articles/2016-09-30/summers-floats-idea-of-sustained-government-stock-purchases, accessed 12 October 2019.

buy? This seems difficult to imagine as purchases need not be confined to government bonds but could include private credit instruments, equities, housing, even fine art."[10]

In some of these examples, policymakers seem to contemplate purchases of assets on a routine and recurring basis. This is actually stronger than what is required to lend substance to notion of a reflecting barrier, i.e. a likelihood of official purchases of assets (or other forms of support) after a substantial fall in prices.

The general point abundantly illustrated by the above examples is that in an era of low interest rates, such that the zero lower bound is a significant constraint on monetary policy, any past taboo on official intervention in asset markets appears to have been greatly weakened. Consistent with this, it has been widely observed by non-official commentators that starting with the rate cut in response to the stock market crash in 1987, central bankers have become increasingly solicitous about falls in asset prices. Perhaps this is because in a highly financialised economy, the real economic activity and price indices which are targeted by central bank mandates are more heavily influenced by a wealth effect from individuals' portfolio holdings than in the past.

There are also reasons for thinking that rising financialisation, and in particular the rising financial wealth of the middle class, have changed the politics of financial crises so as to favour intervention (Chwieroth & Walter 2019a, 2019b, 2019c). Many voters may now have an increasing awareness of the market value of their pension assets, which was not the case a generation ago, when most pensions were defined benefit. The liberalisation of mortgage lending and other consumer credit over the past 40 years means that voters are also more financially leveraged than in the past. Hence the consequences of asset price falls for the electoral prospects of incumbent policymakers may now be larger than in the past.

Any form of official asset purchase may have effects on the prices of other assets besides those purchased; indeed this is part of the rationale for such a policy. So quantitative easing may support house prices even if not directed explicitly at the housing market. The effect on house prices where houses are not the asset purchased is probably mainly through a yield channel: quantitative easing lowers long term bond yields, and so reduces mortgage interest rates. This was one of the stated rationales of the earliest quantitative easing programmes in the US, which focused on the purchase of mortgage-backed securities, with an explicitly stated intention to "support housing markets".[11]

2.2. Potential future policies after a large fall in prices

[10] 'Bean (2018). .

[11] According to a Federal Reserve press release, 25 November 2008: "This action [purchase of mortgage backed securities] is being taken to reduce the cost and increase the availability of credit for the purchase of houses, which in turn should support housing markets and foster improved conditions in financial markets more generally." https://www.federalreserve.gov/newsevents/pressreleases/monetary20081125b.htm, accessed 20 October 2019.

The discussion above has reviewed government interventions in markets for housing and other assets which have been observed in relatively ‘normal’ conditions after modest falls in prices. For NNEG, we are concerned with policies not in ‘normal’ conditions, but after a large a fall in prices. Here are some forms which such policies could take, both encompassing and extending those previously observed:

– provide subsidies to cover mortgage payments by home owners in financial distress (e.g. like several new schemes introduced by the Ministry of Housing, Communities and Local Government in 2009: the Mortgage Rescue Scheme, the Repossession Prevention Fund, and the Homeowner Mortgage Support Scheme);

– provide financing on favourable terms for purchases of houses (e.g low interest, high loan-to-value, high loan-to-income, long terms, limited recourse, or interest-free equity loans as in Help to Buy);

– provide new tax breaks (e.g. stamp duty cuts, deposit grants, reintroduction of mortgage interest relief for owner occupiers);

– allow individuals early access to their pension funds to fund house purchase (as suggested by the Secretary of State for Housing and Communities in July 2019[12], and the Association of Consulting Actuaries in November 2019[13]);

– provide guarantees of mortgage loans for lenders (e.g. the Help to Buy mortgage guarantee scheme ran from 2013 to 2017 and offered to guarantee up to £12bn of loans; under EU rules, these guarantees had to be charged for, but this might not be necessary in future);

– make regulatory changes to encourage or mandate lenders to relax lending criteria, or to exercise forbearance on delinquent loans (e.g. the ‘Mortgage Pre-Action Protocol’ promulgated by the Ministry of Justice in November 2008);

– buy portfolios of mortgages from lenders, and then relax criteria or exercise forbearance as above;

– nationalise one or more mortgage lenders, and then relax criteria or exercise forbearance as above;

– give temporary guarantees on house prices (cf. the US Treasury’s temporary guarantees on money market mutual funds announced on 19 September 2008);

[12] ‘Brokenshire criticised for suggesting first-time buyers dip into pension’. *The Guardian*, 3 June 2019.
[13] *2019 ACA Pensions and Savings Manifesto.* www.aca.org.uk, accessed 16 November 2019.

- make direct purchases of houses on the open market. This might be rationalised either as an investment with a view to later profitable resale, or as a policy decision to rebuild the stock of social housing towards its mid-20th century levels.

As can be seen from this long list, there is a wide range of potential policies, some substantiated by actions which the State took after 2008. Others may seem outlandish at present; but so would the unconventional monetary policies of the last decade, at any time in the previous 50 years. Unprecedented events – a banking crisis, or a large fall in house prices – have a way of calling forth unprecedented measures. Furthermore, the new exposure of the State to house price movements via equity loans made under Help to Buy, which have terms up to 25 years, may represent a new budgetary incentive for intervention after a fall in prices, distinct from the long-standing electoral incentives.

Contrary to the general picture above of multi-faceted policy support for house prices, there is a strand of opinion which says that house prices are already too high relative to average earnings; and hence that policy should target a long period of stasis in house prices, so that wages catch up and the ratio of house prices to income gradually falls. One example of this strand of opinion is the report *Land for the Many* (Monbiot et al, 2019), a set of proposals on housing prepared for the Labour Party (not currently adopted as policy). But this report places great emphasis on the point that large falls in nominal house prices must be avoided; a whole chapter is devoted to preventing this outcome.[14] Leaving aside the wider merits and demerits of the proposals, the point to note is that even those who advocate radical policies to reduce the long-term rate of growth of house prices nevertheless say that this should be combined with policies to prevent large falls.

It is possible that the preference just expressed, for stable rather than increasing house prices, may gain wider and more fervent currency. If owner occupation continues to decline, perhaps voters in aggregate will care less about falling house prices, and policymakers will also care less. But having observed the social and policy responses to the financial crisis, which to my surprise did not seem to reduce either popular aspirations or policy support for home ownership, I now think that such a reduction is less likely.[15] For better or worse, we live in a country where home ownership is supported by all the major political parties, many MP's own more than one property, senior monetary policymakers assert in unguarded moments that property is a better personal investment for retirement than a pension, etc.[16] The political calculus also includes an endowment effect: a large fall in prices

[14] The proposed solution is the 'Common Ground Trust', a State entity which would buy the land under owner-occupied houses and lease it back to the occupier. It is argued that purchases by the Common Ground Trust could counteract any fall in demand for land from builders consequent on other proposals in the document, and so stabilise land (and hence house) prices.

[15] In the 2018 British Social Attitudes survey, 87% of respondents said that they would prefer to buy rather than rent their home. This proportion has remained broadly stable in previous surveys over the previous 30 years. https://www.gov.uk/government/collections/public-attitudes-to-housing-british-social-attitudes-survey, accessed 8 December 2019.

[16] 'Property is better bet than a pension says Bank of England economist' *The Guardian*, 28 August 2016.

causes current home owners to suffer concentrated and actual losses, whilst others (e.g. current renters who might purchase in future) receive only diffuse and hypothetical gains. Consequently, home owners are not only more numerous than renters, they are also more motivated and organised; they take a greater interest in local planning control, and they are more likely to vote.[17] Policymaker ideologies and preferences can of course slowly change, when new generations of MPs and officials take office. But any change is likely to be gradual, because the ideology of policy support for house prices runs deeper than the political hue or personalities of any particular administration.

To summarise this section: there is a long history of policymaker intervention in the housing market, including in recent years policies such as Help to Buy which directly raise prices. There are also some reasons to think that monetary policymakers in future may be more sensitive than in the past to falls in the prices of risk assets, and particularly to house prices. The prospect of such intervention does not place a completely firm barrier under house prices. Nevertheless, it does raise prices relative to those which would prevail in the absence of the intervention, and a reflecting barrier is one way of representing this effect.

3. Alternative justifications for a reflecting barrier

In the previous section the reflecting barrier was conceived as a crude representation of the likelihood of future policymaker intervention after a large fall in house prices. The present section considers some alternative rationales for assuming a floor under house prices, higher than the zero floor which is appropriate for equity in a single company. In the NNEG context, we are primarily concerned with differences in the *lower tail* of long-term returns from houses compared with equities. Limiting falls in the lower tail will also typically have the effect of raising *mean* returns, but the effect on mean returns is not our primary concern.

Throughout this section I refer to *land* rather than *houses*. Land is to be understood as freehold land. A leasehold potentially introduces different considerations, analogous to the pull-to-par effect for a bond, particularly if the lease is short or the ground rent is high; but houses considered for equity release are usually freehold (or very long leasehold, which is nearly equivalent to freehold). A freehold house price notionally comprises two parts: land value and construction costs (indexed to current prices). The considerations I discuss in this section principally affect land values. According to estimates from the Office of National Statistics, 72% of the value of the total stock of dwellings in the UK is attributable to land values (2015 estimates).[18] Similarly, Knoll et al (2017, p348) estimate that 73% of the increase in UK house prices between 1950 and 2012 was attributable to land values, and the rest to construction costs. The 73% figure is the lowest from the 14 countries considered; the

[17] According to the polling organisation Ipsos, in the December 2019 election, 70% of outright owners and 64% of mortgage holders voted, compared with 51% and 52% of private and social housing tenants. https://www.ipsos.com/ipsos-mori/en-uk/how-britain-voted-2019-election, accessed 22 December 2019.

[18] Data from https://www.ons.gov.uk/economy/nationalaccounts/uksectoraccounts, accessed 26 November 2019.

mean is 84%. Hence the considerations I discuss in this section for land values can also be expected to be large factors in house prices.

The discussion focuses on the following considerations:

- Freehold land is an absolute claim, but equity is a residual claim
- The supply of new land is less elastic than the supply of new equity
- Land in good neighbourhoods is a positional good.

3.1 Freehold land is an absolute claim, but equity is a residual claim

The equity of a company incorporated with limited liability is legally defined as the residual value after all creditor claims, with a lower limit of zero. Even for a company with substantial current value, it is straightforward to envisage future circumstances in which the equity would unambiguously be worth zero. The company might not be leveraged at present, but it could easily take on new secured debt in future. If the debt cannot be repaid in line with its terms, the debtholders will take over the firm, and its original equity will then then worth zero. Given this fundamental nature of equity – a residual claim, with a lower limit of zero – a model where the price can go arbitrarily close to (but not below) zero is reasonable.[19]

Freehold land, on the other hand, is an absolute claim, not a residual claim. There is no possibility of its ever being wiped out by unrepayable debt. Freehold land might become worthless if it falls into the sea, or suffers some comparable annihilation; but for most plots of land, these circumstances are more remote than equity being wiped out by unrepayable debt. Freehold land might also become worthless if the legal system through which ownership claims are defined and enforced collapses; but in this state of anarchy, an asset delineated by physical boundaries such as walls would probably retain more value than claims delineated only on paper, such as equities and bonds. In summary, the circumstances required for previously valuable freehold land to become worthless, whilst not impossible, seem considerably more remote than for equity.

The much greater ease of borrowing secured against a house rather than other assets (as discussed in Section 2.1.1 above) supports the observations in the present section. Banks are more willing to lend secured on land rather than equity because of the different nature of the underlying claims. It might be

[19] Note that the lower limit of zero on equity is not a matter of logic but of convention, where the convention is currently substantiated by the legal construct of limited liability. It is conceivable that equity could be re-defined to have a lower limit below zero. For example, the 'Stop Wall Street Looting Act' proposed by Senator Elizabeth Warren would impose joint and several liability on private equity funds for the debts of companies in which they hold equity. .

argued that a more proximate driver of banks' preferences is regulation, in particular the very low risk weights assigned to loans secured on houses compared with those secured on other assets. This may be true, but it raises the question: why is regulation like that? The answer is the same: the different nature of absolute claims represented by land values versus residual claims represented by equity values makes the former more suitable as collateral.

3.2 The supply of new land is less elastic than the supply of new equity

When investor demand for equity in a particular industry increases, the price of equity rises. This tends to induce a supply response: it becomes more attractive for private companies to list, or for entrepreneurs to enter the industry and issue new equity. This supply response is straightforward and elastic, and it limits the rise in price of equity in response to the increased demand.

On the other hand, if demand for land for house-building increases, the possibilities for a supply response are less straightforward. New land for house-building can be created only by re-zoning the planning status of agricultural or industrial land. But this re-zoning is heavily constrained by political calculus: the potential losses from re-zoning (to existing homeowners in a locality) are concentrated, but the potential benefits (to hypothetical purchasers or tenants of houses not yet built) are dispersed. Whether political obstacles to re-zoning may increase or decrease in future is a matter for reasonable debate. But even if planning restrictions were substantially relaxed, the creation of new residential land seems likely to remain constrained, contested and slow compared with the creation of new equity, which represents the everyday functioning of capital markets.

There are other assets besides land with a relatively inelastic supply response to increasing demand: this pattern prevails to some degrees for commodities such as coal, oil, metals, etc. However, for most commodities, efficiency of extraction and efficiency of use have tended to increase over time. There has been relatively little increase in the efficiency of use of land for houses. The main possible mechanisms for increased efficiency of use are building higher into the sky, and improvements in transport efficiency. Building higher is tightly constrained by planning restrictions, and ultimately by tastes and technology. On transport efficiency, if it were to become feasible to commute rapidly over long distances, the price of houses close to major urban centres, where network effects lead to clustering of high-paying jobs, might decline. But there has been little reduction in commuting times over the past half-century, and at present there is no sign of the technological breakthrough in transport efficiency that would be needed to significantly reduce house prices near urban centres.[20]

[20] Miles and Sefton (2018) note that there were large improvements in transport efficiency from the early 19th century up until the mid-20th century (e.g. invention of railways, motor cars, aircraft), which may explain the limited rise in land values over this period. But improvements since then have been negligible, which may have contributed to the steep rise in land values.

3.3 Land in good neighbourhoods is a positional good

Land in good neighbourhoods is in part a positional good. That is, the utility derived from the good depends on how much of it one possesses *relative to how much others possess*. If you possess a positional good, you receive positive utility, and those who do not possess it receive negative utility as a consequence of your possession of it; a positional good is essentially zero-sum. Land as space (a material good) is *physically* scarce; but land in whatever are deemed to be good neighbourhoods (a positional good) is *socially* scarce.

As society becomes richer, basic needs for food and shelter can be satisfied by a smaller fraction of income. Also, the cost of producing many goods tends to fall in real terms, because of technological advances, and competition between suppliers ensures that these cost reductions are passed on in the prices of finished goods. But positional goods cannot be manufactured; they derive from social comparison, and so the combination of technological advance and supplier competition has no effect on their prices. Therefore the fraction of society's aggregate income spent on competing for positional goods tends to rise with increasing wealth (Turner 2016, p70; Stiglitz 2015, p4). This underpins land values in good neighbourhoods.

It is conceivable that social competition might gradually be redirected towards *other* positional goods. If it became fashionable for every family to own the best boat they can afford, rather than the best house they can afford, the positional value of land in good neighbourhoods might decline. But this fanciful possibility seems much more remote than the processes of technological change and competition which can undermine the prices of non-positional goods (and indirectly, the equity of companies which produce those goods).

3.4 Antecedents in classical economics

Each of the above points – the absolute nature of freehold claims, the inelasticity of supply of land, and the positional nature of land – may substantiate a floor for land values (and hence house prices) somewhere above the floor of zero which seems appropriate for equity in a single company. Fundamentally, freehold land is a different type of asset from equity, with *a priori* reasons for less density in the lower tail of long-term returns. None of the points is fully convincing on its own, but in combination they may be persuasive, particularly if viewed through the lens of investment epistemology (rather than mathematical epistemology). By investment epistemology, I mean the view that it is better to have several weak (but at least partially independent) reasons for believing something, rather than one purported rigorous 'proof'. Rigour seldom changes anyone's mind about anything important in economics, and nor should it, because all rigour is local.

It is interesting to note that the distinctive nature of land was well recognised by classical economists such as Adam Smith (1776), David Ricardo (1817) and John Stuart Mill (1848). The distinctive nature of land was also politically salient in the second half of the nineteenth century, particularly in the United States, where the social reformer Henry George developed an economic philosophy which became known as Georgism. This philosophy advocated that people should own the value they create themselves; but that the economic value derived from land (over and above the cost of bringing it into production), which derives from the land's position rather than the owner's efforts, should be shared by all society. The principal policy prescription of Georgism was an annual tax on land values. George's ideas were enormously popular in his lifetime; his text *Progress and Poverty* (George, 1879) sold over three million copies, probably more than any earlier book by an American author on any subject. However, the distinctive properties of land were later de-emphasised by neoclassical economists starting with John Bates Clark (1888), who posited just two main factors of production, labour and a fungible concept of 'capital', with land being merely one form of capital. This conflation of land and capital in neoclassical economics may have been encouraged by political considerations: the concept of a land value tax was unattractive to landowners, who were often influential benefactors of university departments and economic research (Gaffney, 1994).

4 Intuition for NNEG value in the presence of a reflecting barrier

In this section I give some initial intuition on how a reflecting barrier affects the value of NNEG.

4.1 Simple example

I start with a simple illustrative example. I ignore mortality and other decrements (which are not salient to the option pricing aspects of NNEG) and instead assume a NNEG written today for a 65 year old customer has a term certain of 25 years. Other assumptions are

- house spot price, $S = 1$ (equivalently: we work in units of the spot house price);
- initial loan-to-value ratio = 0.3;
- roll-up rate = 4% (so strike price of option, $X = 0.3 \times 1.04^{25} = 0.8$);
- risk-free rate, $r = 1.5\%$;
- deferment rate, $q = 1\%$; and
- house price volatility, $\sigma = 13\%$.

Some points in these assumptions merit further explanation. The *deferment rate*, a term originated by the PRA, is the discount rate which when applied to the spot price of the house gives the *deferment price*, that is the price payable now for possession of the house at the end of the term. In other option pricing literature, the deferment price is normally called the prepaid forward price. The initial loan-to-value of 0.3, roll-up rate of 4% and option term of 25 years are convenient examples inspired by

Turnbull (2019a), although clearly the term in particular will vary widely across the different years summed for even a single customer's NNEG. The house price volatility of 13% and deferment rate of 1% correspond to the minimum parameters suggested by the PRA in Consultation Paper 13/18, before the PRA moved to periodic updating of these parameters. These suggestions are again merely convenient examples; the pattern of results does not depend on using the minimum parameters, as we shall see when we flex them later.

On the above assumptions, the Black-Scholes style price[21] for a put option is 0.077, and a call option is 0.306. Buying a call and writing a put involves an outlay of $C - P = 0.229$; by put-call parity, we pay a premium of 0.229 now to enter a forward contract to buy at 0.8 after 25 years.[22]

The premium to enter a forward can equivalently be calculated as the present value of the forward price (i.e. the price agreed today for payment after 25 years to receive the asset at that time), *less* the present value of our actual obligation to pay 0.8 after 25 years; or symbolically, $e^{-rT}[Se^{(r-q)T} - 0.8S] = 0.229$ as expected.

We shall refer to the equivalence just noted between $(C - P)$ and the premium to enter a forward with the same strike price as the 'standard put-call parity'. It can be shown that using risk-neutral prices for both puts and calls is mutually implicative with the standard put-call parity (e.g. Derman & Taleb, 2005). The 'standard' qualifier is necessary in this paper because we shall find that in the presence of the reflecting barrier, this mutual implication breaks down. The next section starts to give some intuition for this.

4.2 Introducing a reflecting barrier

Now assume a new commitment from a policymaker to intervene to support the price at a lower barrier $b = 0.8$ (i.e. the same as the strike, i.e. the rolled-up loan for our example NNEG). This means that if the price falls to the level of the barrier, the policymaker makes a small purchase so that price reflects instantaneously off the barrier, spending no finite time there. Assume the commitment is fully credible to all. What difference does this make?

- Market participants recognise the new commitment to intervention, so the spot price rises, say from S to S'. The forward price rises in the same proportion, from $Se^{(r-q)T}$ to $S'e^{(r-q)T}$. This is

[21] Actually calculated using the Black '76 formula, Equation (8) later in the paper.

[22] As a reminder for the reader: put-call parity says that if we buy a call and write a put with the same maturity T and strike price X, this is economically equivalent to entering into a forward contract to purchase the stock by paying X at time T. This is because if the stock price ends up above X, we exercise our call; if the stock price ends up below X, the put is exercised against us; so we are exposed to all the upside and downside at expiry, and we are committed to paying X at expiry. So the net cashflow for buying the call and writing the put must be equivalent to the premium for entering into the forward contract.

because interventions move spot and forward in the same way, and the cash-and-carry arbitrage which determines forward pricing operates in the same way as before. So the forward price expr*essed in a numeraire of the (new) observed spot price* remains 0.229 (but the forward price expressed in £ is higher than before).

– The put now costs us nothing to buy (since the fully credible barrier means the price can never go below the strike). Note that Black-Scholes gives a different (higher) price, which is clearly not sensible.

– Buying the call, on its own, now appears equivalent to entering a forward contract to buy at 0.8. This is because the call now must have a positive payoff (since the barrier prevents the price ever going below 0.8), and so we are certain to exercise it. So applying the logic of standard put-call parity suggests that the price of a call should equal the 0.229 premium to enter a forward (but we shall see later that this is not the only plausible concept of price for a call). Note that Black-Scholes again gives a different (higher) price.

– As a consequence of the previous two points, the link between using Black-Scholes prices for both puts and calls and the standard put-call parity seems to be broken. This breakage is perhaps unsurprising for standard Black-Scholes (which clearly has no term for barrier), but we shall see later that the breakage is more fundamental, and persists even when we construct a risk-neutral option price which allows for the barrier.

– One can start to get some intuition for this breakage as follows. Without the barrier, continuous variation in the assumed rate of growth for the underlying asset leads to continuous variation in the prices of each of puts and calls. The risk-neutral rate is the *only* assumed rate of growth which satisfies the standard put-call parity for all possible strikes X (Derman and Taleb, 2005). But with a reflecting barrier equal to the strike, a put is clearly worth zero for *any* assumed rate of underlying asset growth! This apparent indeterminacy hints that the link between a specific assumed rate of growth (the risk-neutral rate) and put-call parity may break down.

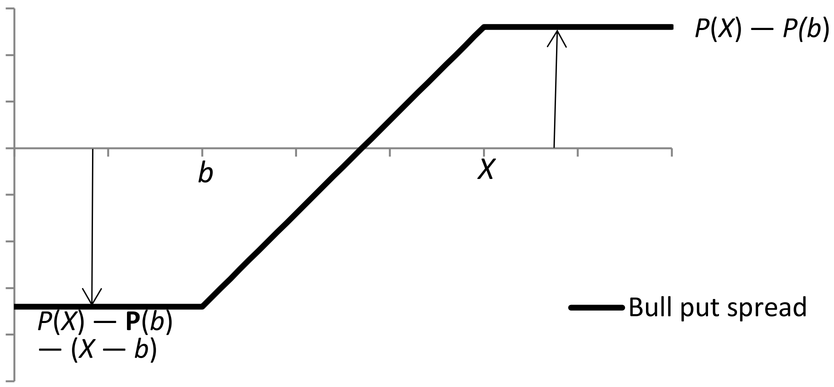
In summary, ordinary Black-Scholes doesn't give sensible valuations in the presence of a reflecting barrier, and there seem to be complications with the standard put-call parity.

4.3 Bull put spread as an upper bound

A bull put spread is a well-known option trade, which can be used to get an upper bound for the value of a put in the presence of a reflecting barrier. The trade involves selling a put with a high strike (in our case, the NNEG strike X, i.e. the rolled-up loan), and buying a put with a low strike (in our case b, the level of the barrier). The payoff diagram for the trade is shown in Figure 1. Note that the maximum loss we might incur at maturity is (X – b), but the price diffusion on the path to maturity can go below b (and may be below b at maturity). With a reflecting barrier, the maximum loss is the same, but the diffusion is modified so that it can never go below b. Intuitively, this must reduce the value of a put compared with the bull put spread, because in scenarios where the diffusion previously went below b, it is now reflected off the barrier. So the bull put spread $P(X)-P(b)$ represents an upper bound on the value of NNEG with a reflecting barrier.

The next section considers how we might improve on this upper bound.

Figure 1: Payoff diagram for Bull put spread



5 Risk-neutral valuation formula for NNEG in the presence of a reflecting barrier

This section gives a formula to price a put option with a reflecting barrier under the spot price, using the risk-neutral pricing paradigm. A formula for a call option on a stock without dividends in the presence of a barrier was originally developed by Veestraeten (2008), and that for the corresponding put by Hertrich and Veestraeten (2013). Hertrich (2015) and Hertrich & Zimmermann (2015, 2017) give a formula for a put on an exchange rate with a reflecting barrier. Their formula involves two risk-free interest rates, one in the transaction currency and one in the settlement currency. The adaptation to the NNEG context involves replacing the settlement currency risk-free rate with the deferment rate, that is the discount rate which when applied to spot price gives the price payable now for possession of the house at the end of a term. The adaptation can be intuitively understood by observing the following correspondence:

– The exchange rate is the number of units of the transaction currency (which earns the risk-free rate) required to buy one unit of the foreign currency (which earns *the foreign risk-free rate)*

– The house price is the number of units of the transaction currency (which earn the risk-free rate) required to buy one unit of housing (which earns a net rental yield, assumed equivalent to the *deferment rate*).

The equivalence just assumed between net rental yield and deferment rate depends on the assumption that both the spot price and the deferment price are fully determined by a 'dividend discount' model applied to rents, with the same effective discount rate used by both spot and deferment purchasers. Since the spot price is determined primarily by owner occupiers, but the (hypothetical) deferment price would be determined by a quite different type of purchaser – those interested in deferred possession after many years – it seems unlikely that the two discount rates would be the same. Hence the deferment rate may not be equivalent to net rental yield. For example, the PRA's prescribed minimum deferment rate is set by reference to factors other than net rental yields. In the PRA's view, the deferment rate is best thought of the long-term real risk-free rate, plus a risk premium required by investors in housing, less expected real capital growth, with the latter two items assumed stable over time (PRA Policy Statement 19/19 para 2.6(ii)). If this or some other 'non rental yield' concept of deferment rate is preferred, the adaptation of the currency option formula can alternatively be understood by observing the following correspondence:

– The foreign currency risk-free rate is the rate which gives the prepaid forward price of *1 unit of foreign currency* (expressed in units of the foreign currency today)

– The deferment rate is the rate which gives the prepaid forward price of *1 unit of housing* (expressed in units of housing today, which are equivalent to units of currency, because house prices are always denominated in currency).

I now give a brief sketch of the formal method of derivation. For technical details see Appendix A, and the papers referenced therein.

We start with a standard geometric Brownian motion for the price process. We then impose a reflecting barrier somewhere below the lower of the spot price and the strike price (i.e. $b < \min(S, X)$). The strike can be either lower or higher than the spot price, i.e. in or out of the money. The spot price evolves as a geometric Brownian motion (GBM), except that if the spot price hits the barrier from above, reflection occurs instantaneously, and with infinitesimal size. We can think of this as the State making a small purchase which prevents the price falling below the barrier.

The instantaneous nature of the reflection means that the price does not spend any finite time at the barrier, so no arbitrage opportunities are created (we can never buy at the barrier with certainty of a price rise). The absence of arbitrage in our model of the asset returns ensures that an equivalent martingale measure exists, in other words, a risk-neutralised version of the density for the real asset return exists. The risk-neutralised density for this "reflected geometric Brownian motion (RGBM)" has previously been published. So we can price a put option by integrating its payoff over this risk-neutral density.

The resulting risk-neutral valuation formula for a put P_B with a reflecting barrier at b < min (S, X) is

$$
\begin{aligned}
&P_B(X \mid S, r, q, \sigma, T, b) \\
&= X\,\mathrm{e}^{-rT}\,\Phi\left(-z_1+\sigma\sqrt{T}\right) - Se^{-qT}\Phi\left(-z_1\right) \\
&\quad - b\,\mathrm{e}^{-rT}\,\Phi\left(-z_3+\sigma\sqrt{T}\right) + Se^{-qT}\Phi\left(-z_3\right) \\
&\quad + \frac{1}{\theta}\left\{\begin{matrix} be^{-rT}\Phi\left(-z_3+\sigma\sqrt{T}\right) \\ -Se^{-qT}\left(\dfrac{b}{S}\right)^{1+\theta}\left[\Phi\left(z_4\right)-\Phi\left(z_2\right)\right] \\ -Xe^{-rT}\left(\dfrac{X}{b}\right)^{\theta-1}\Phi\left(z_2-\theta\sigma\sqrt{T}\right)\end{matrix}\right\}.
\end{aligned}
\tag{1}
$$

which can be understood as

$$\boxed{P_B\left(X\right) = P\left(X\right) - P\left(b\right) + Adjustment} \tag{2}$$

where the two P on the right-hand side are the Black '76 prices for puts with strikes X and b (i.e. calculated using Equation (8) below), and *Adjustment* is the $1/\Theta$ term

with

X = strike price (for NNEG, the projected rolled-up loan amount at maturity), S = spot price, T = term, b = barrier price, r = risk-free rate[23], q = deferment rate, σ = volatility, Φ(.) is the standard Normal cumulative distribution function

and

$$z_1 \;=\; \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S}{X}\right)+\left(r-q+\frac{\sigma^2}{2}\right)T\right] \tag{3}$$

[23] In its Effective Value Test, the PRA prescribes r as 'the published Solvency II basis risk-free rate for maturity T, adjusted for use on a continuously-compounded basis' (SS 3/17, para 3.20). There may be a case for discounting at a higher rate including an illiquidity premium to reflect the illiquid nature of NNEG, but I leave that aside in this paper.

$$z_2 \quad = \quad \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{b^2}{XS}\right)+\left(r-q+\frac{\sigma^2}{2}\right)T\right] \tag{4}$$

$$z_3 \quad = \quad \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S}{b}\right)+\left(r-q+\frac{\sigma^2}{2}\right)T\right] \tag{5}$$

$$z_4 \quad = \quad \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{b}{S}\right)+\left(r-q+\frac{\sigma^2}{2}\right)T\right] \tag{6}$$

$$\theta \quad = \quad 2\frac{(r-q)}{\sigma^2} \tag{7}$$

For a zero barrier (b = 0), the formula reduces to the standard Black '76 formula

$$P(X \mid S,r,q,\sigma,T) = \mathrm{e}^{-rT}\left[X\Phi\left(-z_1+\sigma\sqrt{T}\right)-Se^{(r-q)T}\Phi\left(-z_1\right)\right] \tag{8}$$

and for a barrier equal to the strike *(b = X)*, the formula gives a value of 0. This confirms that the formula works as expected at both extremes.

It is important to note that the formula in Equation (1) applies only for a barrier less than the lower of the spot price and strike price ($b < \min(S,X)$). The requirement $b < S$ is straightforward and can be can be ensured by coding a cap of S on the input parameter b. The requirement $b < X$ requires some care, because the barrier is fixed for valuing options over all terms, butt a NNEG is evaluated by summing a series of 'mini-NNEGs' over terms of 1,2,3…years; so for low loan-to-value cases, the shorter terms may well have $b > X$. For these terms, the 'mini-NNEG' is clearly worth zero, but naïve application of the formula will wrongly give a positive value. So as a precaution in any practical implementation, the formula should be embedded in a conditional statement which returns the correct zero value if $b > X$.

Although the formula looks complicated compared with Black '76, the complication is algebraic rather than parametric: there is only a single extra parameter, the barrier level b. The value of the single parameter can be thought of as a shorthand for different strength of beliefs about the prospects for intervention, or about the different nature of housing assets compared with equity assets as discussed in Section 3 above. Whilst the barrier feature is an artificial model of intervention, it is also easy to describe and explain, including for audiences who may not understand all the niceties of option pricing.

Note that as with Black ’76, the formula does not include any term relating to the expected rate of growth of house prices. The price process is modified at the barrier, but otherwise it has the same drift as ordinary GBM, and this drift is removed by risk neutralisation.

Note also that the $P(X)-P(b)$ part of the formula is the premium we receive to enter a *bull put spread* as described in Section 4.3. As discussed in that section, this represents an upper bound on the value of NNEG with a reflecting barrier. So the *Adjustment* is zero for $b = 0$ and $b = X$, and negative everywhere in between. For the examples in this paper, the magnitude of the *Adjustment* is largest for a barrier somewhere above 0.5x the strike. At this level of barrier and with other parameters as in the example in Section 4.1, the *Adjustment* is up to half the value of the bull put spread, so there is a substantial difference.

The formula gives a risk-neutral price for a put. The risk-neutral price of a call with the same strike X would normally be immediately inferred from put-call parity (i.e. call = forward + put). However some care is needed with this in the presence of the reflecting barrier. Contrary to the usual pattern, the call price inferred in this way is *not* the same as a risk-neutral price for a call. Since we are mainly interested in the price of a put for NNEG, a pragmatic solution is to adopt the risk-neutral price for the put, and note the need for caution about inferring a price for a call. Fortunately we do not normally require call prices in the context of NNEG, nor are there any market prices for forwards and calls with which we need to ensure consistency. The technical point is discussed further in Appendix B.

One interpretation of a risk-neutral option price is that it represents the cost of replicating the payoffs of the option by continuous hedging. Although we cannot in practice do this for NNEG (because houses cannot readily be sold short), it is interesting that the hedging recipe turns out to be simpler than casual inspection of the put formula might suggest. This is covered in Appendix C.

The formula can be reconciled to the Gerber and Pafumi (2000) formula for the value of a dynamic guarantee on an investment fund. This is covered in Appendix D.

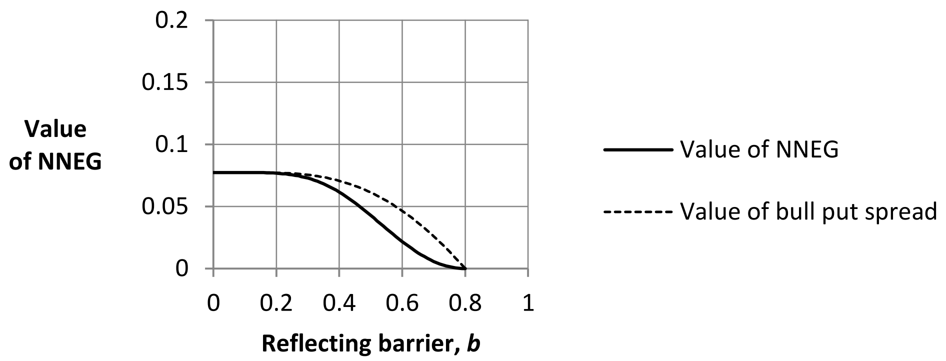
6 Examples and sensitivity tests

6.1 Examples

I now consider a series of examples to show how a reflecting barrier affects the risk-neutral valuation of NNEG. Figure 2 shows the value of the example NNEG specified in Section 4.1, for all possible levels of the reflecting barrier (recall that the option strike is $0.3 \times 1.04^{25} = 0.8$, so any non-trivial barrier must be below 0.8). The solid curve shows the value of NNEG with all parameters as in section 4.1 (notably deferment 1%, volatility 13%). The dashed curve shows the corresponding bull put spread. The value of NNEG is equal to the bull put spread at $b = 0$ and $b = X$, and less everywhere in between, as expected.

It is important to remember that in all our formulae and graphs, the numeraire is the observed spot house price, *given the barrier level in force.* As noted in Section 4.2, a rise in the level of the barrier generally assumed by market participants will lead to a rise in the observed spot price; but all NNEG values are then expressed in units of the new observed spot price. So we should be cautious about applying the formula with the *old* spot house price to assess the effect of introducing or changing a barrier. In the NNEG case, this distinction is of little import, because we are not contemplating the 'introduction' of a barrier; the barrier (if it exists) has always been there, or at least has been there for a very long time. But the distinction may be important in other applications.

Figure 2: NNEG with reflecting barrier compared with a bull put spread



In the remainder of this paper, I adopt the parameters from Section 4.1 as the central assumptions (i.e. corresponding to the solid curve in Figure 2), and show the effect of equal step changes up and down for each key parameter around these central assumptions.

Figure 3 shows the sensitivity when volatility is flexed up or down 5%. Note that for lower levels of volatility, the curve is nearly flat over a wider range. This is because for lower levels of volatility, the risk-neutral density has less mass at low prices, so that it makes almost no difference whether the barrier is at 0 or 0.5; only a barrier quite close to the strike makes a difference.

The ordering of the three curves reverses towards the right of Figure 3 (albeit this cannot be seen clearly given the scale of the graph). In other words, for a barrier very close to the strike, a rise in volatility slightly *reduces* the value of a put. Intuitively, this is because when the barrier is very close to the strike, a rise in volatility cannot materially increase the intrinsic value of the option, and does make it more likely that the spot price will end up just above rather than just below the strike (meaning that the option expires worthless). But this curious effect is mainly of theoretical interest, because a barrier very close to the strike makes the option nearly worthless anyway. This effect applies only for a put option; for a call option, the effect of volatility is monotonically positive, as expected.

Figure 3: Sensitivity to volatility

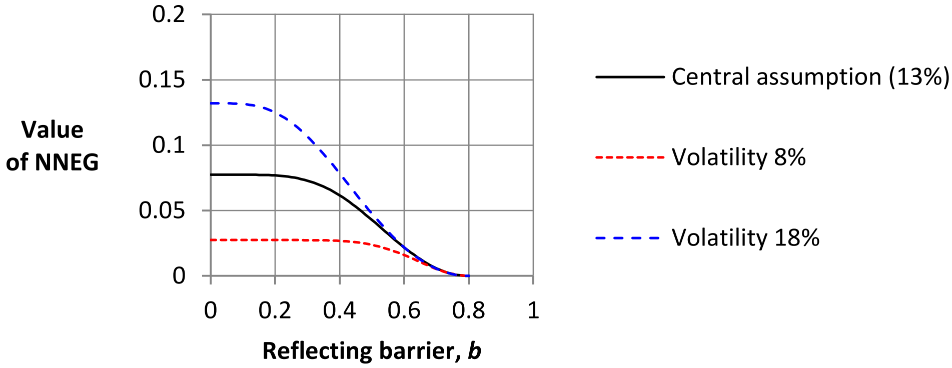


Figure 4 shows the sensitivity when the deferment rate is flexed up or down 1%. A higher deferment rate raises the value of NNEG.

Figure 4: sensitivity to deferment rate

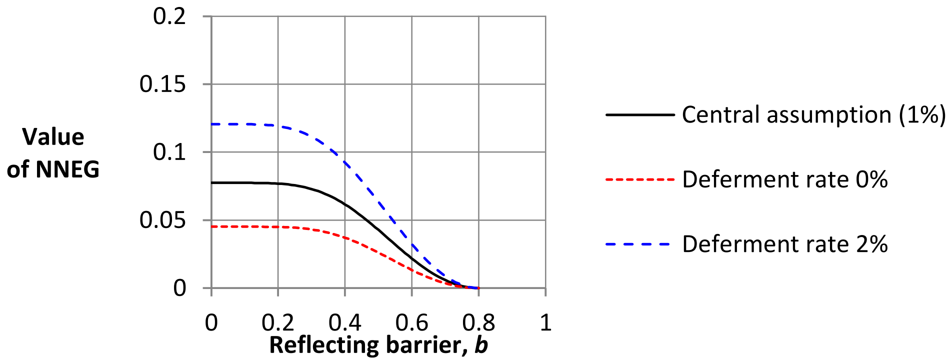


Figure 5 shows the sensitivity when the discount rate (risk-free rate) is flexed up or down 1%. A higher discount rate reduces the value of NNEG. The shape and dispersion of the lines in this graph are very similar to the previous graph for the deferment rate, but the blue and red lines are interchanged. This is because deferment rate and discount rate have analogous but opposite effects on the value of NNEG. Increasing the deferment rate reduces the 'deferment price' input into the formula; increasing the discount rate applies more discounting to the option payoff.

Figure 5: sensitivity to discount rate

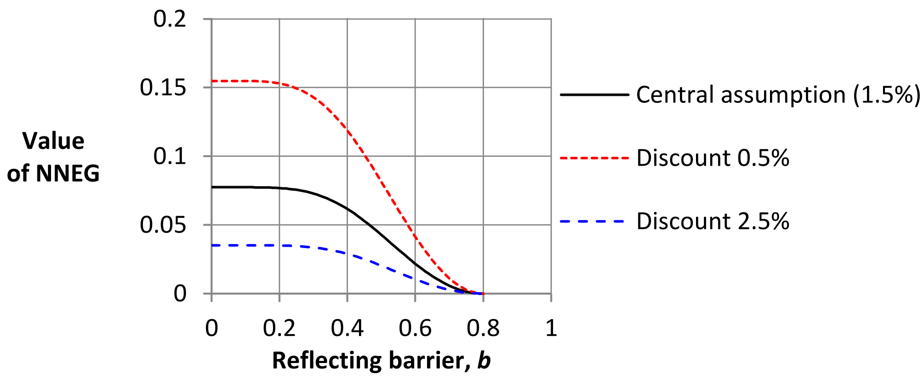
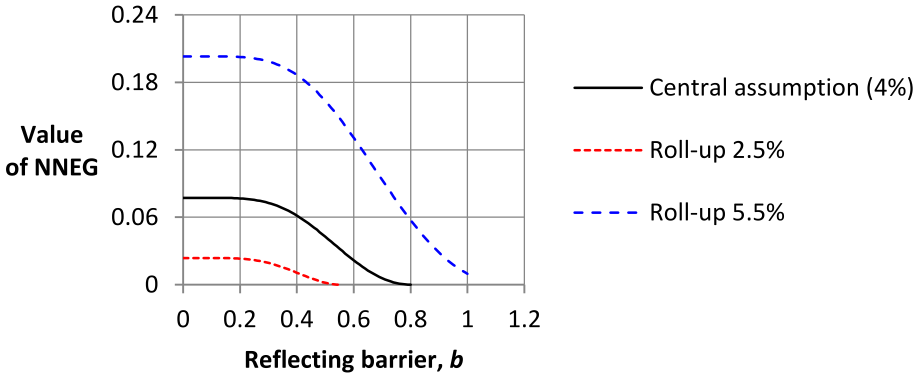


Figure 6 shows the sensitivity when the roll-up rate on the NNEG is flexed up or down 1.5%. Increasing the roll-up rate increases the value of NNEG. This is because given the assumed 30% initial loan-to-value and 25 year term, roll-up rates of 2.5%, 4% and 5.5% correspond to strikes of 0.556, 0.800 and 1.144 respectively. For the strike of 1.144, the curve terminates at 1 because the maximum feasible barrier is the current spot price of 1. Note that a barrier of say 0.6 reduces the value of NNEG by 100% for 2.5% roll-up (because the loan never rises above the barrier), 70% for 4% roll-up, and 35% for 5.5% roll-up. In other words, the fractional reduction (but not the absolute reduction) is larger for smaller cases.

Figure 6: sensitivity to roll-up rate



Flexing the term certain or initial loan-to-value of the NNEG unsurprisingly has similar effects to flexing the roll-up rate (recall that strike = initial loan-to-value times (1+ roll-up rate)^T), so no separate graphs are shown for these. The variations which give equivalent effects to those shown for 2.5% and 5.5% in Figure 6 are term certain flexed to 16 years and 34 years (from central assumption of 25 years), or initial loan-to-value flexed to 0.21 and 0.43 (from central assumption of 0.30).

6.2 Comparison with 'real-world' valuations

A common but disputed approach to NNEG valuation in practice has been to make an *ad hoc* adjustment to the Black '76 formula in Equation (8), replacing the forward price of the house with a projected price of the house at maturity. In other words, the *forward rate* ($r - q$) is replaced by an assumed *house price growth rate* (say g). The assumed g is invariably higher than ($r - q$), and so this gives a lower answer than Equation (8); furthermore, it is usually higher than the risk-free rate r on its own, and so implies a *negative* input for the deferment rate q in Equation (8). This is sometimes called the 'real-world' method (e.g. Hosty et al, 2008, paragraph 7.7.4), and it produces values dramatically lower than Black '76. How does the barrier model compare?

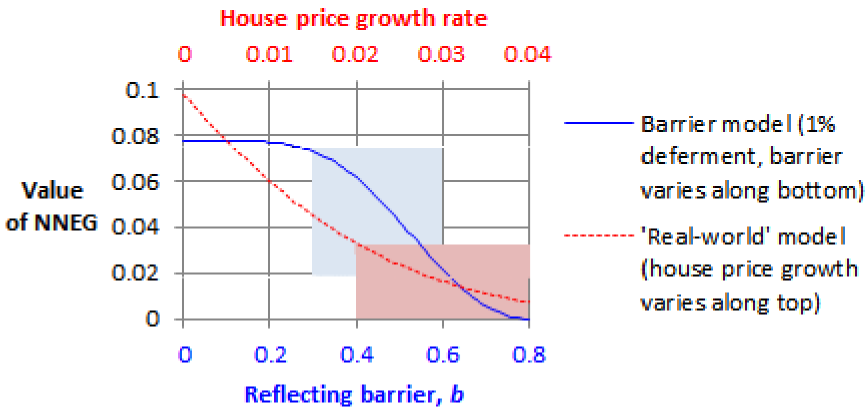
Table 1: comparison of barrier model (strike price = 0.8) with 'real-world' model

Barrier model		'Real-world' model	
Barrier level	**NNEG value**	**House price growth**	**NNEG value**
0	0.0774	0%	0.0977
0.2	0.0768	1%	0.0599
0.4	0.0616	2%	0.0332
0.6	0.0217	3%	0.0166
0.8	0	4%	0.0073

In Table 1, the left-hand side shows NNEG values using the barrier model, for a range of barrier levels. The right-hand side shows NNEG values using the 'real-world' method, for a range of house price growth rates. On both sides, all other parameters are set to the same central assumptions as in the previous graphs and section 4.1.

The comparison between the two methods is more fully illustrated in Figure 7. The blue solid curve represents values from the barrier model, with the variable barrier level on the lower axis. The red dashed curve represents values from the 'real-world' model, with the variable house price growth rate along the upper axis. The blue shaded area indicates a range of what might be regarded as plausible assumptions for the barrier, say 0.3 to 0.6. The red shaded area indicates a range of what might be regarded as plausible 'real world' assumptions for house price growth, say 2% to 4%. The overlap of the two shaded areas is small, indicating that the barrier method produces higher valuations than the 'real-world' method for most plausible assumptions. This comparison will apply *a fortiori* for any higher assumed strike than 0.8 (with a higher strike, the effect of a barrier at any given level diminishes, so the NNEG value from the barrier model rises closer to the Black '76 value).

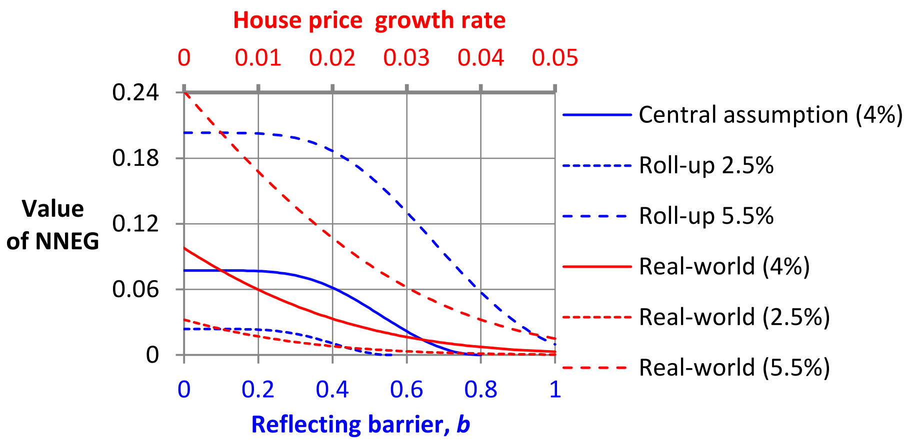
Figure 7: Comparison of barrier model (variable barrier, bottom axis) with 'real-world' model (variable house price growth, top axis)



Notwithstanding this comparison, it is important to note that the rationales of the barrier and 'real-world' methods are different. The barrier method uses a reflecting barrier, which implies a different stochastic process for the price, but otherwise applies the standard principles of risk-neutral valuation. The 'real-world' method uses an *ad hoc* modification of Black '76, for which it may be difficult to provide specific justification. The difference in rationales is reflected in the different shapes of the curves in Figure 7. In the 'real-world' method, an increase in the assumed house price growth rate gives a convex reduction in the value of NNEG, for all levels of the house price growth rate. In the barrier method, the effect of an increase in the assumed barrier level depends how close the barrier already is to the strike.

The barrier model implies a zero valuation for any NNEG where the rolled-up loan at maturity is less than the level of the barrier (albeit this zero value needs to be coded by embedding the valuation formula in a conditional statement, as mentioned in Section 5). More generally, the barrier model's valuation as a multiple of a corresponding 'real-world' valuation will tend to be higher for higher roll-up rates, higher initial loan-to-value ratios, and longer terms to maturity. For roll-up rates, the pattern is illustrated in Figure 8. The greater separation of the red and blue curves as we move up the graph shows that as we increase the roll-up rate, the barrier valuation gets further above the 'real-world' valuation. Specifically, the top pair of red and blue curves shows that for 5.5% roll-up, any level of barrier up to about 0.7 gives a higher valuation than the real-world method with typical house price growth assumptions (say 2% – 4%). On the other hand, the bottom pair of red and blue curves show that for 2.5% roll-up, any barrier above 0.42 gives a lower valuation than the 'real-world' method with house price growth of 2%; and any barrier above 0.55 gives a zero valuation (because the rolled-up loan never exceeds the barrier).

Figure 8: Barrier model versus 'real-world' model: sensitivity to roll-up rate



7 What level of barrier to use?

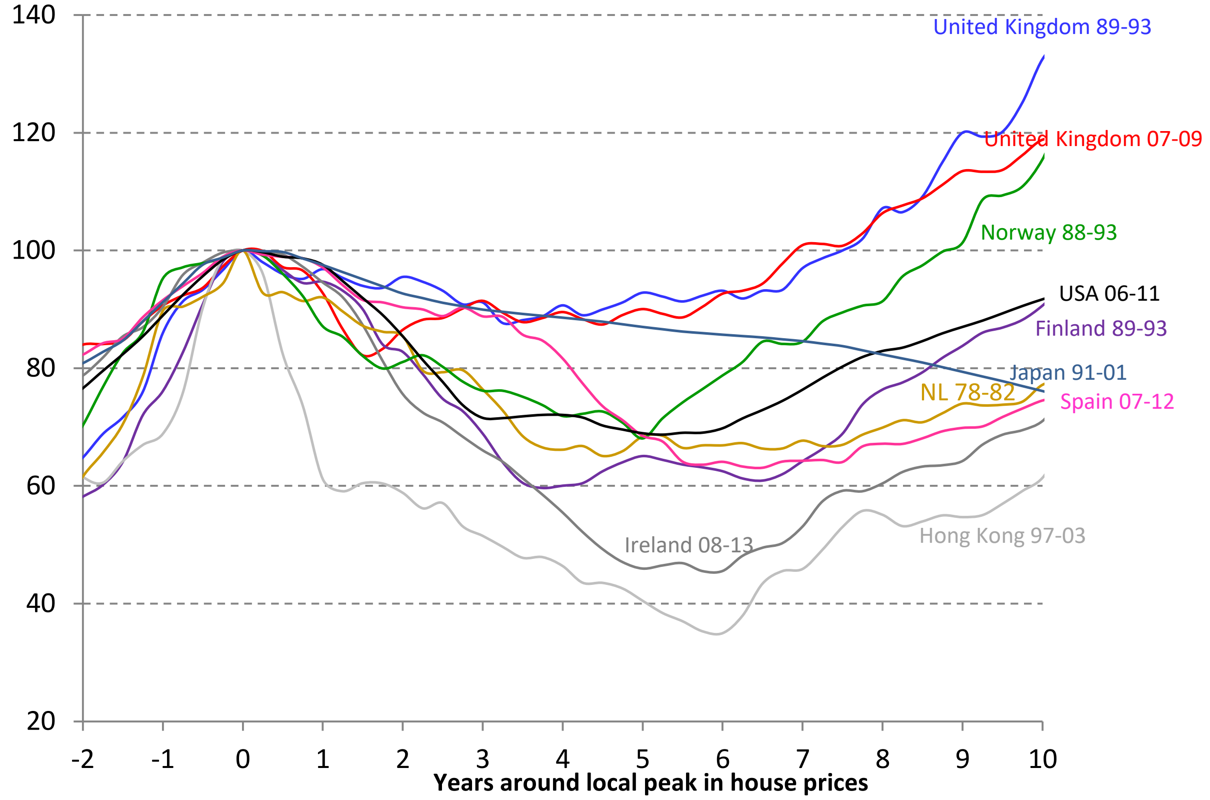
In the slightly different context of designing stress tests, Jeffery and Smith (2019, p47) suggest the following aphorisms:

- If something has happened before, it can happen again
- If it has happened elsewhere, it can happen here
- If it happens, when it happens it will happen faster than last time.

Whilst the purpose is slightly different, these aphorisms seem worthy of consideration when choosing the level of a reflecting barrier. The barrier is a hypothetical construct intended to represent either the prospect of future interventions or the distinctive nature of housing (and especially freehold land) assets, as discussed in Section 2 and 3 above; but a barrier which has actually been breached in the past may seem suspect. At the same time, my view is that the aphorisms are best regarded as guidelines rather than absolute rules.

Figure 9 shows the paths of indices of nominal house price indices from 2 years before to 10 years after a local peak, for selected countries and periods. These are not typical housing market downturns; the selection is intended to illustrate some of the *worst* recorded slumps in house prices in a range of countries with similar levels of economic development to the United Kingdom. The data are drawn from quarterly indices for nominal house prices collated by the Bank for International Settlements (BIS), re-based to 100 at the local peak. The methods of calculation of the various underlying indices are unlikely to be fully comparable across countries, but the BIS seems to be the best available source of curated data for a wide range of countries.

Figure 9: national house price indices around some of the largest falls in developed countries



Data source: https://www.bis.org/statistics/index.htm

I make following observations on Figure 9 and related data.

- The biggest falls in the United Kingdom have been modest compared to some other countries: national indices showed peak-to-trough falls of 12% in 1989-93 and 18% in 2007-09, according the BIS (which for the UK, draws on the Office of National Statistics indices, which are based on actual transaction prices at the Land Registry). It is also notable that these falls were followed by much stronger recoveries at 10 years than in any of the other countries. Perhaps the United Kingdom has been lucky. Or perhaps the polity of the United Kingdom is such that intervention after falls in house prices is more readily forthcoming than in other countries.

- These modest reported falls represent national indices, but the prices of individual houses (some of which may fall much more) are the relevant factor for NNEG.

- The modest reported falls in the United Kingdom also mask considerable regional variation. The BIS figures are drawn from national indices published by the Office of National Statistics, which are based on Land Registry records for all purchases. Other organisations publish regional sub-indices, based on their proprietary samples of purchases funded by a mortgage. The Halifax House Price regional sub-indices recorded rises for Scotland and the north of England during the 1989-93 national slump, but also a largest regional fall of 35%

from peak to trough for East Anglia. In the 2007-09 slump, the fall in the Halifax national index was 21% from Q3 2007 to Q1 2009 (comparable to the BIS figure of 18%), with the largest regional fall (over roughly the same period) being 26% in Greater London.

- Hong Kong 1997-2003 and Ireland 2008-2013 represent exceptionally severe slumps, where the peak-to-trough falls over about six years exceeded 50%, but both with strong recoveries shortly thereafter.

- For scale reasons, Figure 9 does not show the full decline in Japan, where prices fell 46% over 18 years from the peak in 1991 to the trough in 2009. In Q2 2019, some 28 years after the peak, Japanese house prices were still 37% below their 1991 level.

All things considered, I tentatively suggest a value of 0.5 as a suitable level for the reflecting barrier in a United Kingdom context. Note that this encompasses the exceptional long-term experience in Japan. I choose to elide the reported falls below 50% in Ireland and Hong Kong, for three reasons. First, the periods below –50% were brief, with strong recoveries shortly thereafter. Second, 0.5 is a convenient round figure. Third, the range of possible policy intervention in Ireland and Hong Kong may have been constrained by their inability or unwillingness to print their own currencies (Ireland uses the Euro, and the Hong Kong Dollar is pegged to the US Dollar). Others may prefer to use lower or higher values for the barrier, which are simple to implement in Equation (1) in Section 5.

A barrier level of 0.5 would lead to values for NNEG always lower than Black ’76, and often significantly lower, as can be seen in the graphs in Section 6. For a given level of barrier, the fractional reduction in the value of NNEG compared with Black ’76 is smaller for higher roll-up rates, higher initial loan-to-value, and longer terms (these all increase the rolled-up loan and hence strike price, so the barrier becomes less significant). The barrier method’s higher relative valuation of the more leveraged cases might be regarded as a reassuring feature.

8 Discussion

8.1 Potential applications of the barrier model

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The potential policymaker interventions in Section 2 and the alternative rationales in Section 3 cannot substantiate a completely firm barrier under house prices. Nevertheless, they do tend to limit the severity of price falls relative to those which would prevail in the absence of the intervention or other rationales, and a reflecting barrier is one way of representing this effect. Possible uses for the barrier model include reserving, investment analysis, risk transfer, and assessing trade-offs in public policy, as follows.

8.1.1 Reserving

If permitted by regulation, the barrier model could be used for reserving. This is discouraged by current United Kingdom regulation, where the PRA's Effective Value Test explicitly specifies use of the Black '76 formula to value NNEG (see section 1.2 above), and so disregards the possibility of policy intervention. Acknowledging the possibility of policy intervention may be a delicate matter for a prudential regulator, for three reasons. First, most plausible forms of intervention involve monetary policy and fiscal policy, which are outside the purview of a prudential regulator. Second, a realistic official acknowledgement of the likelihood of intervention after a calamitous fall in prices may have the unwanted side-effect of encouraging excessive risk-taking. Third, we are concerned with the prospects for intervention over a long timescale, well beyond that of any particular government or policy regime. I have no easy solutions to these difficulties, but nevertheless I think that simply ignoring the prospect of policy intervention is not realistic. And even if policy intervention is to be ignored on principle, the alternative rationales in Section 3 above merit consideration.

8.1.2 *Investment analysis*

Where the State does not give any lead through reserving regulation, an investor may nevertheless have a view that if house prices fall a long way, some form of policy intervention is likely. The barrier model can assess how much difference this makes to share valuations.

8.1.3 *Risk transfer*

The barrier model can also be used in pricing bulk risk transfers between parties who share the belief that some level of policy intervention is likely after a large fall in prices; or for product pricing, capital allocation, and other management purposes.

8.1.4 *Assessing trade-offs in public policy*

The current regulatory requirement that insurers must make allowance for NNEG using a model where much of the NNEG value derives from very extreme scenarios for house prices reduces the assessed value of insurers' assets. This implies that more assets are required to back liabilities (for NNEG writers, principally annuities), which in turn implies that annuity pries are higher than they would otherwise be. Without making a pre-commitment to intervene at any particular level of prices, the State may nevertheless wish annuities to be available on terms which exclude the possibility of house prices falling below a certain level. This is not a denial of the possibility that prices might fall to even lower levels; it is an expression of (reasonable) preferences about allocations over different future states of the world. Stated differently, it is a judgment that society prefers to deal with some extreme contingencies *ex-post*, rather than by mandating that resources are set aside *ex ante*.

This trade-off between affordable annuity pricing and requiring insurers to reserve for very extreme scenarios *ex ante* is a social and political choice. It is not a decision that should be made solely by a prudential regulator. In debate about NNEG valuation, there seems to have been little explicit discussion of this trade-off, perhaps because it has been difficult to quantify. The barrier model provides one potential way of quantifying the trade-off. For example, we could investigate how much difference an assumed barrier at each of several different levels makes to insurers' costs of capital, and hence to annuity pricing.

8.2 Effect of the barrier on volatility and the spot price

The model in this paper treats intervention as an extraneous event which is which is *superimposed* on the price process, but not *anticipated* by the price process. This is most straightforwardly applicable in a scenario where the barrier represents a guarantee provided on a specific investment fund (e.g. Gerber & Pafumi, 2000; Imai & Boyle, 2001), which is not relevant to anyone other than investors in the fund, and so cannot affect the spot price. However in this paper, the barrier is relevant to market participants in general. In this scenario, the model provides a mathematically tractable approach, but it may seem unrealistic in the following respects.

(i) First, if market participants anticipate the intervention, we might expect volatility (or at least its downside component) to fall as the spot price approaches the barrier.

(ii) Second, if the value of a put option today is reduced by the prospect of intervention at the barrier, we might also expect today's spot price to be raised by the existence of the barrier.

On the first point, long-term volatility is normally estimated by scaling short-term volatility (i.e. $\sigma_T = \sigma\sqrt{T}$). Provided that the barrier has been well below the spot price throughout the observation period used to estimate volatility, the existence of the barrier far below the spot can reasonably be expected to have only a negligible effect on observed short-term volatility. Hence scaling this should produce an estimate consistent with the volatility parameter as defined in the model.

On the second point, it turns out that this is not problematical, given the way we have expressed option prices in this paper. In all the examples in Section 4.1 and Section 6, the observed spot price was set to 1; so our barrier, all strikes and all option prices are *expressed in a numeraire of the observed spot price*. Suppose the spot price is £100 and a fully credible barrier is newly introduced at £90. We might expect the spot price to rise considerably, say to £120; but in the framework of this paper, we would then observe a barrier not at 0.9 but at 0.75. The pricing formula still works if the numeraire is a spot price elevated by a universal barrier (as in the NNEG scenario, and in the set-up of the derivation in Appendix A), rather than a spot price unaffected by a guarantee provided on a single fund (as in the Gerber and Pafumi (2000) scenario).

This argument relies on the particular choice of the spot house price as our numeraire. If for any reason this is not done, then in the NNEG context there are two further points that can be made. First, we are not contemplating the 'introduction' of a barrier and consequent change in the spot price; the barrier has always been there (or at least, has been there for a considerable time). Second, the barrier is not at 90% of the spot price but at a much lower fraction, and so if its existence does raise the observed spot price (in a numeraire of £), the effect is likely to be small. In particular, it should be proportionally much smaller than the effect of the barrier on the £ value of a deep out-of-the-money put option, which is drastically reduced (or even made worthless, if the barrier is above the strike).

8.3 More realistic barrier concepts and ESG models

The concerns discussed in the previous section turned out on closer examination to be less serious than they first appeared. A more compelling criticism is that the fixed nature of the barrier (i.e. a fraction of the spot price S at $T = 0$) for options of all terms is unrealistic. It seems unlikely that policymakers considering intervention at some distant future date will make their decisions by reference to today's price level. They are more likely to be concerned with how far and how fast prices have fallen from their recent levels at the time when intervention is being contemplated. So the barrier model fails to represent the (plausible) prospect of interventions conducted after falls from higher prices than today, and in this respect it under-states the effects of intervention. This is particularly relevant for NNEG, because the strike price increases for longer terms (i.e. a NNEG is evaluated by summing the product of option valuation and exit probability over all possible terms, with the longest terms having strike prices which may be well above the house price today).

A more realistic barrier concept would be the barrier for an option of term T to be a fraction of the highest point on the price path between 0 and T (i.e. a fraction of the high-water mark over the option term). Intuitively, the effect of such a barrier on the value of NNEG would be larger than in this paper. Unfortunately a NNEG with a path-dependent barrier seems more difficult to value. NNEG values in this type of model would probably have to be valued by simulation, i.e. an Economic Scenario Generator (ESG) approach. With this approach, more complex models of intervention could be considered, depending not just on the magnitude of the fall in prices from an initial value or a high-water mark, but also on the rate of change (e.g. intervention occurs if prices have fallen more than 15% in one year, or 30% in three years).

As an alternative to assuming intervention according to a particular rule or rules, an ESG model might allow indirectly for the effects of intervention in the structural form of the price process. This could explicitly model a change in volatility (and potentially also drift) when house price growth deviates significantly from its long-term mean. One reasonable model may be to assume that with increasing deviations of the rate of house price growth from its long-term trend, policymakers become increasingly likely to intervene. This could be represented by a mean-reverting model for house price growth, where the level of mean reversion is itself mean-reverting.

8.4 Idiosyncratic property risk

The NNEG is an option on an individual property, not a property index. An objection to the reflecting barrier model, or indeed to any model of policymaker intervention, is that interventions will be targeted at the general level of house prices, not individual properties. Plausible interventions may not preclude any particular property suffering idiosyncratic local effects (e.g. planning blight, lack of maintenance, flooding) which cause it to fall to a very low price. It is true that a barrier model cannot provide a literal representation of these very few cases. But there also seems no reason to think that the lower tail of an unrestricted geometric Brownian motion provides a literal representation of these very few cases.

9 Conclusions

This paper has modelled NNEG with a reflecting barrier under house prices. The primary rationale for the barrier is that it represents the prospect of policymaker intervention after a large fall in prices. Alternatively, the barrier can be viewed as a way of making allowance for the different nature of underlying housing (and particularly freehold land) assets in NNEG valuations, compared with the underlying equity assets in many other option valuations. Freehold land is an absolute claim, but equity is a residual claim; the supply of new land is less elastic than the supply of new equity; and land in good neighbourhoods is a positional good. None of these points is fully convincing on its own, but in combination they may constitute a persuasive case for assuming a reflecting barrier somewhere above the zero floor that is appropriate for equity assets.

Realistic interventions cannot place a completely firm barrier at a particular level, and in this sense the model may overstate the certainty and efficacy of intervention. On the other hand, the model also understates the prospect of interventions where prices first rise substantially from today's levels and then fall; and it ignores the possibility that market participants' anticipation of intervention will change the price process (and in particular the volatility near the barrier). In short, limits to the realism of the model are not difficult to find. Nevertheless, it may still represent an improvement on extant standard models. The latter make no allowance at all for intervention in a market where most voters hold most of their wealth, which seems to me very unrealistic, and naïve about political economy.

Although the NNEG valuation formula with a reflecting barrier may appear complicated compared with Black ’76, the complication is algebraic rather than parametric: there is only a single extra parameter, the barrier level *b*. The value of the single parameter *b* can be used as a shorthand for different strength of beliefs about the prospects for intervention, or about the different nature of housing assets compared with equity assets. Whilst the barrier feature is an artificial model of intervention, it is also easy to describe and explain, including for audiences who may not understand all the niceties of option pricing.

I have tentatively suggested a value of 0.5 for the level of the reflecting barrier as a fraction of today’s spot price. This would lead to values for NNEG always lower than Black ’76, and often significantly lower. For a given level of barrier, the fractional reduction in the value of NNEG compared with Black ’76 is smaller for higher roll-up rates, higher initial loan-to-value, and longer terms. The barrier method’s higher relative valuation of the more leveraged cases might be regarded as a reassuring feature.

Others may have different views on an appropriate level for the barrier, or disagree that we should use a barrier at all. If no barrier is used, this paper highlights that much of the resulting value of NNEG may derive not from poor scenarios where house prices merely stagnate or fall, but from extreme scenarios where they fall by 50% or more over the term of the NNEG.

Appendix A

Derivation of the put option pricing formula

We start from the observed price S_0, which is a price *given* the long-standing existence of the barrier. The derivation then makes use of two price processes: a process for the observed price in the presence of the barrier, and a process for the ‘shadow price’ of an asset with the same starting value S_0 and same drift and variance, but where barrier has no effect.

We assume that the shadow price follows a real-world price process of geometric Brownian motion, specified as usual by

$$dS_t = \mu S_t d_t + \sigma S \, dW_t \tag{9}$$

where μ is the drift, σ is the diffusion and dW_t is the increment of a standard Wiener process.

Assuming the underlying pays a continuous yield q, the risk-neutral process for the ‘shadow price’ is obtained by replacing the drift μ with $(r - q)$:

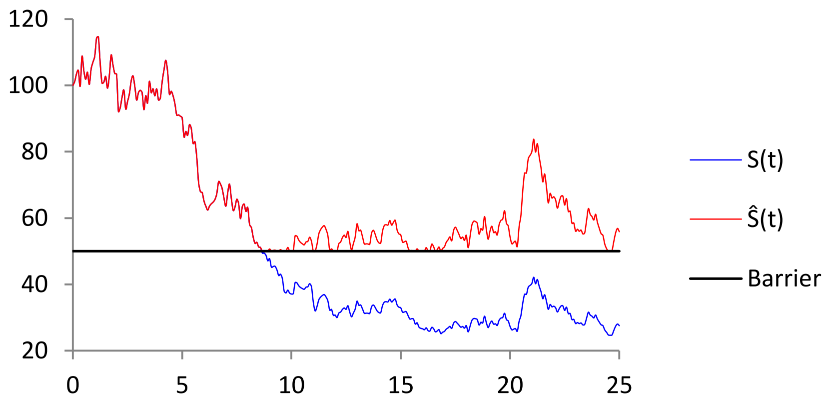
$$dS_t = (r-q)S_t d_t + \sigma S\, dW_t \qquad (10)$$

Now consider the reflecting barrier b, such that the price is reflected instantaneously at the barrier. Because of the barrier, the observed process will be

$$\tilde{S}_t \quad = \quad S_t \cdot \max\left[1, \max_{0\le s\le t}\left(\frac{b}{S_t}\right)\right] \qquad (11)$$

In words: the process $\tilde{S}_t$ is equal to S_t, if S_t has never gone below the barrier; or an up-rated version of S_t which 'undoes' the maximum proportional deficit relative to the barrier, if it has. Figure 10 illustrates how it works, for one unfavourable simulation over 25 years.

Figure 10: one simulation for S_t and $\hat{S}_t$



Now scale this observed process $\tilde{S}_t$ by dividing by the barrier b (so the barrier becomes 1), and then take logs (so the barrier becomes 0). Then the resulting process $\ln\left(\tilde{S}_t/b\right)$ is equivalent to a process $\ln\left(S_t/b\right)$, but with a barrier introduced at zero. We call this process 'reflected geometric Brownian motion' (RGBM). By a standard construction in the theory of stochastic processes, RGBM can be specified by adding an extra term to Equation (10), as follows:

$$dS_t = (r-q)S_t d_t + \sigma S_t dW_t + S_t dL_t \qquad (12)$$

where the process L_t is a 'reflection function' as in Skorokhod (1961, p266). L_t starts from $L_0 = 0$ and increases whenever S_t hits the lower barrier of zero. L_t keeps track of the cumulative amount of upward reflection. This reflection at the barrier is instantaneous, and of infinitesimal magnitude. Hence the observed process does not spend finite time on the barrier, and there are no jumps. It

Together, these features ensure that the no-arbitrage property is preserved; in other words, it is never possible to buy at the barrier for a certain profit with no risk.

The RGBM process as specified in Equation (12) is a semi-martingale, because it can be decomposed into a local martingale (the Wiener term) and two finite variation processes, the drift and the reflection. So we can apply Itô's lemma to Equation (12), to obtain

$$d\ln\left(S_t\right)=\left(r-q-\frac{\sigma^2}{2}\right)d_t+\sigma dW_t+dL_t \tag{13}$$

Conveniently, the density of the RGBM process is known (Veestraeten, 2004). The following formula gives the risk-neutral probability of reaching the price S_T at maturity, evaluated at time 0 when the spot price is S_0, with b as a reflecting barrier (Hertrich 2015, p235; or without the income, Veestraeten 2008, also p235):

$$\begin{aligned} f\left(S_T\right) &\equiv f(S_T \mid S_0,r,q,\sigma,T,b) \\ &= \frac{\left\{ e^{-\frac{\left|\ln\left(\frac{S_T}{S_0}\right)-\left(r-q-\frac{\sigma^2}{2}\right)T\right|^2}{2\sigma^2 T}} + e^{\frac{\ln\left(\frac{b}{S_0}\right)\left[2(r-q)-\sigma^2\right]}{\sigma^2}} e^{-\frac{\left|\ln\left(\frac{S_T S_0}{b^2}\right)-\left(r-q-\frac{\sigma^2}{2}\right)T\right|^2}{2\sigma^2 T}} \right\}}{\sigma S_T\sqrt{2\pi T}} \\ &\quad -\frac{\left[2\left(r-q\right)-\sigma^2\right]}{S_T\sigma^2} e^{\frac{\ln\left(\frac{S_T}{b}\right)\left[2(r-q)-\sigma^2\right]}{\sigma^2}} \\ &\quad \cdot\left\{1-\Phi\left(\frac{\ln\left(\frac{S_T S_0}{b^2}\right)+\left(r-q-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)\right\} \end{aligned} \tag{14}$$

This reduces to the density of GBM for the case $b = 0$, as expected.

The risk-neutral price of the put with the reflecting barrier can then be found by integrating the intrinsic value of the put over this risk-neutral density. That is, we integrate

$$P_B\left(X\right)=e^{-rT}\int_b^X\left(X-S_T\right)f(S_T)\,dS_T \tag{15}$$

to give the valuation formula in Section 5 of the paper (except that the S_0 denoting the current spot price in the density in (14) has been elided to just S for brevity in Section 5.)

The reader may wonder why after neutralising the drift at Equation (10), we subsequently turned our attention to the reflecting barrier, but then did nothing to neutralise the interventions at the barrier. Is something missing? One way of understanding this is to note that the interventions represent a form of 'hybrid return', neither fully risk-free nor fully stochastic. The *incidence* of the interventions if the price touches the barrier is guaranteed, but the *quantum* of the interventions over any given term is uncertain. Note also that because of the instantaneous and infinitesimal nature of the interventions, their effect cannot be hedged away. This explains why terms for the barrier remain in the risk-neutral valuation formula, rather than disappearing in the same way as the term for the (hedgeable) drift in the underlying.

Sources

Veestraeten (2008) derives the risk-neutral price for a call option on a non-dividend paying asset with a reflecting barrier under the price. As an after-thought, he obtains a price for a put option from the standard put-call parity; but this is problematic, because using risk-neutral prices for both puts and calls is not consistent with forward pricing in the presence of the reflecting barrier (see Appendix B). Hertrich and Veestraeten (2013) give the correct formula for the risk-neutral value of a put on a non-dividend paying asset. Hertrich & Zimmmeman (2017) consider a put on an exchange rate, which involves two risk-free rates, one in the transaction currency and the other in the settlement currency. This version is the one directly adapted for the present paper. Hertrich (2015) gives a fuller account of the derivation, leading to the same formula but with the terms grouped in a slightly different way, and also gives a more technical discussion of the put-call parity issue than my intuitive treatment in Appendix B. The working paper Hertrich & Zimmerman (2015) gives some of the necessary integrals.

Appendix B

Put-call parity in the presence of a reflecting barrier

The put option formula in the main text looks correct at the extremes and plausible everywhere in between. It was obtained by turning the handle on the usual risk-neutral pricing machinery. But if we do the same for a call option with barrier less than strike ($b < X$), we obtain another plausible looking formula:

$$
\begin{aligned}
&C_B(X \mid S, r, q, \sigma, T, b) \\
&= Se^{-qT}\Phi(z_1) - X\,\mathrm{e}^{-rT}\,\Phi\left(z_1 - \sigma\sqrt{T}\right) \\
&+ \frac{1}{\theta}\left\{ \begin{aligned} &+Se^{-qT}\left(\frac{b}{S}\right)^{1+\theta}\Phi(z_2) \\ &-Xe^{-rT}\left(\frac{X}{b}\right)^{\theta-1}\Phi\left(z_2 - \theta\sigma\sqrt{T}\right) \end{aligned} \right\}
\end{aligned} \tag{16}
$$

Note that this is the Black '76 formula for a call, plus the $1/\theta$ term.[24]

The difference of the risk-neutral values for call and the put (Equations (16) less (1)) is

$$
\begin{aligned}
&= Se^{-qT}\Phi(z_3) - Xe^{-rT} \\
&+be^{-rT}\left(1 - \frac{1}{\theta}\right)\Phi\left(-z_3 + \sigma\sqrt{T}\right) \\
&+\frac{1}{\theta}\left\{ S_t\,\mathrm{e}^{-qT}\left(\frac{b}{S_t}\right)^{1+\theta}\Phi(z_4)\right\}
\end{aligned} \tag{17}
$$

which we shall call the 'risk-neutral put-call parity'. The risk-neutral put-call parity can also be written as

$$
e^{-rT}\left\{\mathrm{E}^Q\left[S_T\right] - X\right\} \tag{18}
$$

where the Q-superscript indicates that the expectation is calculated using the risk-neutral density for RGBM as in Equation (14). Note that under this density, $\mathrm{E}^Q\left[S_T\right] > S.$ Hence for the risk-neutral put-call parity in (18), the following is true:

(1) it is larger than the 'standard put call-parity', that is premium to enter a forward to buy at strike X, which is

$$
e^{-rT}\left\{Se^{(r-q)T} - X\right\}\text{; and} \tag{19}
$$

(2) it reduces to the standard put-call parity only for $b = 0$.

In other words, in the presence of a reflecting barrier, the difference of risk-neutral call and put prices is not equal to the premium to enter a forward contract. For risk-neutral option prices, standard put-call parity appears not to hold.

---

[24] This expression, and also (1) for the put in the main paper and the difference (17) below, all look slightly different to those in Hertrich (2015), because he defines some of the z-variates differently in that paper. His z_3 is my z_2, and his z_4 is my $-z3 + \sigma\sqrt{T}$. I am using the variates as defined in Hertrich & Zimmerman (2017). Showing equivalence involves unpacking the θ term inside the argument of one of the Normal distribution functions.

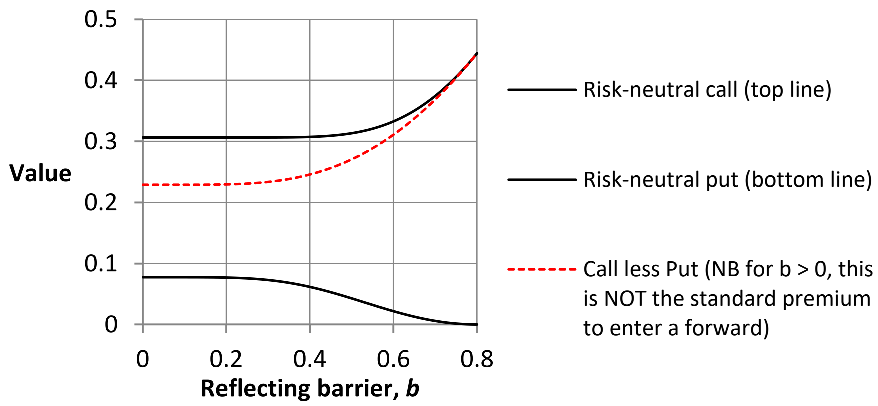
To put some numbers on this phenomenon, Figure 11 illustrates risk-neutral call and put prices and their difference for our central assumption parameters as specified in Section 4.1. Considering each extreme in Figure 11:

- At b = 0 (i.e. no barrier) on the left-hand side, the risk-neutral call (top line) and risk-neutral put (bottom line) prices calculated using Equations (16) and (1) are 0.306 and 0.077; these are the same as the Black '76 values calculated in Section 4.1, as expected. The difference (middle dashed line) is 0.229, which is the same as the premium paid to enter a forward with strike 0.8, again as calculated in section 4.1.

- At b = 0.8 (i.e. barrier equal to the strike) on the right-hand side, the risk-neutral call and put prices calculated using Equations (16) and (1) are 0.444 and 0.

- So the difference of the two risk-neutral option prices (0.444) in the presence of the barrier is larger than premium to enter a forward (0.229). The latter is still calculated by the same formual,a, $e^{-rT}\,[Se^{(r-q)T} - 0.8S] = 0.229$, the same as in the b = 0 case.[25]

Figure 11: Risk-neutral call and put prices with a reflecting barrier



To understand what is happening, we need to think about the different effects that the interventions have on forward pricing and risk-neutral option pricing.

In section 4.2, I said that intuitively, the formula for the price of a forward should not be affected by the barrier, which moves spot and forward in the same way. But how does this intuition reconcile with the risk-neutral expectation $E^Q[S_T]$ in Equation (18)? When we evaluate this expectation using the risk-neutral density in Equation (14), we find $E^Q[S_T] > S$. Does this mean the forward should trade

[25] Note that whilst the algebraic formula for a pricing a forward is unchanged by introduction of the barrier, this does *not* mean that the sterling cost of the premium is unchanged. Recall that all our formulae and graphs use the observed spot price as the numeraire. Introducing a barrier where none existed before will raise the observed spot price ($S' > S$ as discussed in Section 4.2). So the premium to enter a forward using pounds sterling as the numeraire will be higher than before.

higher than $Se^{(r-q)T}$, reflecting the expected 'hybrid return' from interventions? I think not, because if it does trade higher, there in a static arbitrage (short one unit of 'expensive' forward, long one unit of 'cheap' underlying) by which a trader can capture the excess with certainty. The excess should therefore be eliminated by this static arbitrage. In the presence of interventions, the standard arbitrage pricing mechanism for a forward still works, so the normal pricing formula continues to apply.

For options, the effect of the interventions is different. Suppose we try to synthesise the payoff from a put option in the normal way, i.e. by maintaining a portfolio consisting of short a suitable fraction of the underlying asset, long a suitable quantum of a risk-free bond (the relevant quantities are derived in Appendix C). Now consider the reflecting barrier. On any occasion when the spot price touches the barrier, an intervention will occur; *and because the interventions are instantaneous, we cannot adjust the hedge.* So intuitively, both the short position in the asset and the long position in the bond should be smaller, in anticipation of the unhedgeable interventions.

The crucial difference between forward pricing and option pricing is that a forward can be hedged statically, and there is an arbitrage strategy which can capture (and therefore eliminate) any anticipated effect of interventions embedded in the forward price. But option replication requires dynamic hedging, and the hedging strategy cannot respond to the instantaneous reflections at the barrier. Forward pricing eliminates the interventions; risk-neutral option pricing must allow for them. Using risk-neutral option prices (for puts and calls together) is therefore inconsistent with the standard put-call parity.

In these circumstances, what option prices should we use? Recall that as well as (normally) being mutually implicative with the standard put-call parity, risk-neutral option pricing also has another attractive interpretation: it represents the 'cost of manufacture' of the option payoff via dynamic replication. So we can still adopt this interpretation of a risk-neutral price for either of put or call; but if we do so for both,, we shall then be inconsistent with forward pricing. Which should we choose, put or call? There are several reasons for preferring the risk-neutral value of the put:

(a) For NNEG, the 'cost of manufacture' interpretation is of interest for the put, but not of interest for the call. Although we cannot *actually* manufacture a NNEG via dynamic replication, because houses cannot readily be sold short, it may be reasonable to use the *hypothetical* cost of doing so as the value of NNEG (and current PRA regulation encourages this).

(b) The risk-neutral put formula gives a sensible answer at $b = 0$ (where it gives the Black '76 value) and $b = X$ (where it gives 0). Everywhere in between, it is a bit less than the bull put spread, which is consistent with intuition (i.e. the interventions at the barrier make the NNEG a bit cheaper than a bull put spread).

(c) On the other hand, the risk-neutral call formula gives a sensible answer at $b = 0$ (where it gives the Black '76 value), and gives progressively higher values as b rises (as shown in Figure 11). This is consistent with the intuition that anticipating the (unhedgeable) 'hybrid return' from interventions makes a call more costly to replicate. But if we adopt this value for the call and then apply standard put-call parity to infer the put, we get a positive value for the put for $b = X$, which is clearly not sensible, at least for our purpose of NNEG valuation.

(d) More generally, a put is a bounded claim. Any sensible valuation must be in the range $(0, X - b)$, because it can never be worth more than its maximum payoff paid today (assuming positive interest rates). Our formula gives a single price in this range. On the other hand, a call is an unbounded claim, and so can feasibly take a wider range of values.

In the light of these considerations, a pragmatic solution is to adopt the risk-neutral price for the put, and note the need for caution about inferring a call price from the standard put-call parity. Under this approach, the standard put-call parity will imply one price for the call; risk-neutral pricing (or equivalently: the 'risk-neutral put-call parity' in Equation (17)) will imply a different (higher) price. Happily, we do not normally require or observe call prices in the context of NNEG. Nobody can synthesise a bought put by a combination of bought call and forward sale at strike X, because the required instruments are simply not traded. Hence the consideration of put-call parity is more a point of theoretical interest, rather than a need to be consistent with any observed market prices.

More technically, the failure of risk-neutral put and call prices together to ensure put-call parity arises because although the no-arbitrage property is preserved with the reflecting barrier (so a risk-neutral measure exists, and option replication still works), the martingale property is lost. The RGBM process is a sub-martingale under the risk-neutral measure, whereas a martingale must be both a sub- and super-martingale. The high risk-neutral price for the call arises because the intervention creates a non-negative bubble in the risk-neutralised price of the underlying, so there is also a bubble in the price of the call (Heston et al 2007). 'Non-negative bubble' has a technical meaning here, but the ordinary meaning gives the gist.

An intuitive reaction of some readers to this discussion may be that the standard put-call parity is a model-free concept, which option prices ought to satisfy irrespective of the assumed price process of the underlying asset. However, I believe that this intuition breaks down where the price process includes some 'hybrid return' occurring only at a discrete barrier, neither fully risk-free nor fully stochastic. The 'hybrid return' can be captured by the static hedging of a forward, but not by the dynamic hedging of an option. I do not deny that the standard put-call parity is an important concept (and hence this appendix). But I do deny that using risk-neutral prices *for both puts and calls* can be consistent with forward pricing, in the presence of a reflecting barrier as defined in this paper. As Heston et al (2007) succinctly note: "One can choose either put-call parity or risk-neutral option pricing [for both puts and calls], but not both."

Appendix C

Replication strategy

If continuous costless trading in the underlying asset is possible, we can replicate the payoffs of any option which depends only on the terminal value of the asset by maintaining a suitable hedge portfolio (e.g. Baxter & Rennie 1996, p. 95). Although we cannot do this in practice for NNEG (because houses cannot readily be sold short), it is interesting that the theoretical hedging strategy turns out to be simpler than casual inspection of Equation (1) might suggest.

The hedging strategy for a put option involves being short a suitable fraction of the underlying asset and long a suitable quantum of risk-free bond. By continuously adjusting this hedge, we are sure to end up with the 'right' positions in asset and bond to offset the payoffs of the option. For example, if the put option expires in the money, the hedging scheme ensures that we end up short exactly 1 unit of the asset (offsetting the 1 unit of the asset which will be put to us when the option is exercised), and long the bond in an amount equal to the strike price (offsetting the cash we are obliged to pay when the option is exercised).

To find the required fractional short position, we need to find the partial derivative of the put option value in Equation (1) with respect to the asset price *S*. The derivative includes four Normal distribution functions and eight Normal density functions, but remarkably the latter all cancel. Writing *V* for the option value, the required short position in the asset is given by

$$\frac{\partial V}{\partial S} = e^{-qT}\left\{\Phi(z_1)-\Phi(z_3)+\left(\frac{b}{S}\right)^{1+\theta}\left[\Phi(z_4)-\Phi(z_2)\right]\right\} \tag{20}$$

and the required quantum of the long position in the bond is given by Equation (1) less Equation (20).

For $b = 0$, this gives the same hedge fraction and bond investment as for Black '76, as expected. As the barrier increases from zero, the magnitude of both the hedge fraction and bond investment decline, but the latter declines faster, so the value of the hedge portfolio (and hence the option) declines. When b reaches X, both the hedge fraction and bond investment reach zero, again as expected.

Appendix D

Reconciliation to the value of an investment guarantee

Gerber and Pafumi (2000) and Imai and Boyle (2001) value a guarantee on an investment fund. The guarantee is *dynamic* in the sense that whenever the fund falls to the guarantee level, an infinitesimal amount of money is added to the fund to prevent it falling below the guarantee level, and the observed fund value accumulates thereafter with the benefit of that addition. The guarantee is *European* in the sense that the accumulated proceeds of the fund can only be accessed at the maturity date. This combination of dynamic construction and European payoff corresponds to the main features of the options in the present paper. So intuitively, we should be able to reconcile our option formula to their guarantee formula. One way of thinking about this is as follows.

The guarantee implies that the investor receives a fund value with the accumulated benefit of the interventions, but no strike price is payable at the end of the term; and the investor receives the benefit of the interventions irrespective of where the fund value ends up. This payoff can be expressed as a combination of a bought put and a written call *with* the presence of the barrier, both with at-the-money strikes. Note that the two strikes cancel, giving no net payment by the investor at the end of the term, as appropriate for the guarantee. We then need to compare this payoff with that from the same option position *without* the presence of the barrier. The difference in the value of the two payoffs gives the value of the guarantee:

$$C_B(S) - P_B(S) \quad - \quad C(S) - P(S) \tag{21}$$

and expanding this using Equations (1) and (16) with q = 0 (i.e. no income distributed by the fund) then yields Equation 2.11 in Gerber and Pafumi (2000), as expected.

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