INSURANCE LOSS COVERAGE UNDER RESTRICTED RISK CLASSIFICATION: THE CASE OF ISO-ELASTIC DEMAND

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ABSTRACT

This paper investigates equilibrium in an insurance market where risk classification is restricted. Insurance demand is characterised by an iso-elastic function with a single elasticity parameter. We characterise the equilibrium by three quantities: equilibrium premium; level of adverse selection (in the economist's sense); and 'loss coverage', defined as the expected population losses compensated by insurance. We consider both equal elasticities for high and low risk-groups, and then different elasticities. In the equal elasticities case, adverse selection is always higher under pooling than under risk-differentiated premiums, while loss coverage first increases and then decreases with demand elasticity. We argue that loss coverage represents the efficacy of insurance for the whole population; and therefore that if demand elasticity is sufficiently low, adverse selection is not always a bad thing.

KEYWORDS

Adverse selection, loss coverage, risk classification, equilibrium premium, iso-elastic demand.

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1. Introduction

1.1 Adverse Selection and Loss Coverage

Insurance purchased voluntarily is usually underwritten, and premiums are charged depending on individual risk. We call this 'risk-differentiated pricing'. In certain cases, however, insurers may be compelled to charge the same premiums regardless of some factor known to be relevant to the risk. For example, in the European Union since 2012 insurers have been barred from using gender in underwriting. Arguments over the use of genetic information in underwriting began in the mid-1990s and are still going on; many countries have placed limits on insurers' use of genetic test results. The possibility is not merely theoretical.

This paper studies some of the implications of insurers not being allowed to use risk-differentiated pricing. Our starting point is a population partitioned into subgroups by reference to the level of risk of some undesired event, and an insurance company or market ('the insurer') selling simple, standardised insurance contracts, covering the undesired event, to the members of that population.

The actuary's natural response to risk-differentiated pricing being banned is that adverse selection will result. That is, whatever pooled premium is charged, lying somewhere between the 'correct' premiums for the lowest-risk and highest-risk subgroups:

- the premium will appear high to the lowest-risk subgroups, so fewer in those subgroups will buy insurance; and
- the premium will appear low to the highest-risk subgroups, so more in those subgroups will buy insurance.

The resulting losses will force premiums higher, possibly resulting in an 'adverse selection spiral' until equilibrium is reached at a level of premium that is attractive only to the highest-risk subgroups. Actuaries' natural concern is that losses will accrue, though it is entirely possible that an equilibrium will be approached from the other side, if excessively cautious pooled premiums are charged at first.

In this paper, we assume that an equilibrium has been reached and that the insurer is charging 'pooled' premiums, to both high and low risks, that break even. We do not consider how equilibrium was reached, or whether profits or losses were made along the way. We model the insurance market as a timeless equilibrium, 'equilibrium' in the sense that it focuses on the steady state where all insurers' profits and losses are competed away; and 'timeless' in the sense that it glosses over any sequence of profits and losses which occur as insurers adjust the pooled premium towards the equilibrium level. Whilst risk classification is restricted, the level of pooled premiums is not. Because insurers are assumed to adjust the pooled premium to whatever level is necessary to ensure equilibrium, and competition between insurers in risk classification is not permitted, adverse selection does not imply insurer losses.

An equilibrium under adverse selection is often regarded as bad, for several reasons. Fewer low-risk individuals have insurance coverage, while those that remain are subsidising a larger number of high-risk individuals. Also, there is usually assumed to be a reduction in 'gains from trade' since fewer insurance contracts are written.

On the other hand, more high-risk individuals being insured is arguably a social good, since coverage has shifted to where it is most needed. Thomas (2008, 2009) introduced the idea of 'loss coverage', namely the expected claims under a given premium rating scheme at equilibrium. It may be thought desirable, from a social policy point of view, for loss coverage to be as high as possible (depending on the nature of the undesired event). Maximum possible loss coverage would be achieved only if everyone bought insurance, which we assume is not the case. Then adverse selection, by increasing the proportion of high-risk individuals buying insurance, may cause the loss coverage to rise. Since at equilibrium the insurer makes neither profits nor losses, adverse selection could be a social good, despite its name.

This paper follows Thomas (2008, 2009) which illustrated the concept of loss coverage with simulations based on an exponential-power demand function suggested by De Jong & Ferris (2006). This demand function is very flexible, but also rather intractable. Thus, Thomas (2008, 2009) did not give a full mathematical account of loss coverage. Here, we present a comprehensive mathematical analysis of equilibrium premia, adverse selection and loss coverage, based on a more tractable iso-elastic demand function. In doing so, we also give precise definitions of adverse selection and loss coverage, thus highlighting the contrast between the two concepts.

1.2 Literature Review

This paper also follows others which investigate insurance market equilibrium when risk classification is restricted. In the economics literature, recent surveys of such work include Hoy (2006) and Dionne & Rothschild (2014). The work summarised and advanced in these papers typically takes a utility-based approach: individuals make insurance choices to maximise their utility according to some utility function, and the outcomes of different risk classification schemes are evaluated by a social welfare function which is a (possibly weighted) sum of expected utilities over the entire population. For example Hoy (2006) assigns equal weight to the expected utilities of all individuals. In the actuarial literature, Macdonald & Tapadar (2010) also take a utility-based approach, while De Jong & Ferris (2006) instead model insurance demand directly as an elasticity-driven function of the pooled price, without explicitly considering utilities. The present paper follows this last approach.

This paper can also be contrasted with another strand of empirical literature on adverse selection, which focuses on variations in purchasing choices and risk level within the group of insurance buyers, rather than between buyers and non-buyers. See for example Chiappori & Salanie (2000) for auto insurance; Cardon & Hendel (2001) for health insurance; Finklestein & Poterba (2004) for annuities; Finkelstein & McGarry (2006) for long-term care insurance; and Cohen & Siegelman (2010) for a general survey. We adopt a similar metric for adverse selection as in this literature, based on a positive covariance of coverage and losses.

2. Motivating Examples

We now give three heuristic examples of insurance market equilibria to illustrate the concept of loss coverage and the possibility that loss coverage may be increased by some adverse selection.

Suppose that in a population of 1,000 risks, 16 losses are expected every year. There are two risk-groups. Each individual in the high risk-group of 200 individuals has a probability of loss four times higher than that of an individual in the low risk-group. This is summarised in Table 1.

We further assume that probability of loss is not altered by the purchase of insurance, i.e. there is no moral hazard. An individual's risk-group is fully observable to insurers and all insurers are required to use the same risk classification regime. The equilibrium price of insurance is determined as the price at which insurers make zero profit.

Under our first risk classification regime, insurers operate full risk classification, charging actuarially fair premiums to members of each risk-group. We assume that the proportion of each risk-group which buys insurance under these conditions, i.e. the 'fair-premium proportional demand', is 50%. Table 1 shows the outcome, which can be summarised as follows:

(a) There is no adverse selection, as premiums are actuarially fair and the demand is at the fair-premium proportional demand.
(b) Half the losses in the population are compensated by insurance. We heuristically characterise this as a 'loss coverage' of 0.5.

Table 1: Full risk classification with no adverse selection.

	Low risk-group	High risk-group	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums (differentiated)	0.01	0.04	0.016
Numbers insured	400	100	500
Insured losses	4	4	8
Loss coverage			0.5

Now suppose that a new risk classification regime is introduced, where insurers have to charge a single 'pooled' price to members of both the low and high risk-groups. One possible outcome is shown in Table 2, which can be summarised as follows:

(a) The pooled premium of 0.02 at which insurers make zero profits is calculated as the demand-weighted average of the risk premiums: (300 x 0.01 + 150 x 0.04)/450 = 0.02).
(b) The pooled premium is expensive for low risks, so fewer of them buy insurance (300, compared with 400 before). The pooled premium is cheap for high risks, so more of them buy insurance (150, compared with 100 before). Because there are 4 times as many low risks as high risks in the population, the total number of policies sold falls (450, compared with 500 before).
(c) There is moderate adverse selection, as the break-even pooled premium exceeds population-weighted average risk and the aggregate demand has fallen.
(d) The resulting loss coverage is 0.5625. The shift in coverage towards high risks more than outweighs the fall in number of policies sold: 9 of the 16 losses (56%) in the population as a whole are now compensated by insurance (compared with 8 of 16 before).

Table 2: No risk classification leading to moderate adverse selection but higher loss coverage.

	Low risk-group	High risk-group	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums (pooled)	0.02	0.02	0.02
Numbers insured	300	150	450
Insured losses	3	6	9
Loss coverage			0.5625

Table 2 exhibited moderate adverse selection. Another possible outcome under the restricted risk classification scheme, this time with more severe adverse selection, is shown in Table 3, which can be summarised as follows:
(a) The pooled premium of 0.02154 at which insurers make zero profits is calculated as the demand-weighted average of the risk premiums: (200 x 0.01 + 125 x 0.04)/325 = 0.02154.
(b) There is severe adverse selection, with further increase in pooled premium and significant fall in demand.
(c) The loss coverage is 0.4375. The shift in coverage towards high risks is insufficient to outweigh the fall in number of policies sold: 7 of the 16 losses (43.75%) in the population as a whole are now compensated by insurance (compared with 8 of 16 in

Table 1, and 9 out of 16 in Table 3).

Table 3: No risk classification leading to severe adverse selection and lower loss coverage.

	Low risk-group	High risk-group	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums (pooled)	0.02154	0.02154	0.02154
Numbers insured	200	125	325
Insured losses	2	5	7
Loss coverage			0.4375

Taking the three tables together, we can summarise by saying that compared with an initial position of no adverse selection (Table 1), moderate adverse selection leads to a higher fraction of the population's losses compensated by insurance (higher loss coverage) in Table 2; but too much adverse selection leads to a lower fraction of the population's losses compensated by insurance (lower loss coverage) in Table 3. This argument is quite general: it does not depend on any unusual choice of numbers for the examples.

3. The Model

Based on the motivation in the previous section, we now develop a model to analyse the impact of restricted risk classification on equilibrium premium, adverse selection and loss coverage. We first outline the model assumptions and define the underlying concepts.

3.1 Population Parameters

We assume that a population of risks can be divided into a low risk-group and a high risk-group, based on information which is fully observable by insurers. Let μ_1 and μ_2 be the underlying probabilities of loss, of an individual in the low-risk group and high risk-group respectively ($\mu_1 < \mu_2$). Let p_1 be the proportion of the population in the low risk-group and $p_2 = 1 - p_1$ be the proportion of the population in the high risk-group. For simplicity, we assume that all losses are of unit size. All quantities defined below are for a single risk sampled at random from the population (unless the context requires otherwise).

Define L to be the loss in respect of a person chosen at random from the population. L is a random variable, the randomness arising from the existence of different risk-groups, and the fact that a loss may or may not eventuate. The expected loss is given by:

$$E[L] = \mu_1 p_1 + \mu_2 p_2. \tag{1}$$

$E[L]$ corresponds to a unit version of the third row of the tables in Section 2.

Information on risk being freely available, insurers can distinguish between the two risk-groups accurately and charge premiums π_1 and π_2 for risks in risk-groups 1 and 2 respectively. Moreover, individuals themselves know to which risk-group they belong, and in that light they will adjust their demand for insurance according to its price. Define the demand function $d(\mu, \pi)$ to be the probability that an individual, whose probability of loss is μ, will buy one unit of insurance if they are offered it for premium π.

Given an individual picked at random from the population, define the insurance coverage Q as follows: $Q = 1$ if the individual buys insurance, and $Q = 0$ otherwise. Q is a random variable, because the demand function governs only the probability that an individual will buy insurance. The expected insurance coverage is given by:

$$E[Q] = d(\mu_1, \pi_1)p_1 + d(\mu_2, \pi_2)p_2. \tag{2}$$

$E[Q]$ corresponds to a unit version of the fifth row of the tables in Section 2.

Note that we do not assume that individuals within each risk-group are homogeneous, except for their common probability of loss. They may have different characteristics and preferences, which are unobserved, so their insurance purchasing decisions appear to be random. We represent this apparent randomness with this simplest possible probabilistic model, a Bernoulli random variable.

Suppose the insurer charges premium π_1 to individuals in the low risk-group and π_2 to individuals in the high risk-group. Given an individual picked at random from the population, the premium they pay is a random variable, denoted Π, the randomness arising from membership of one or other risk group, and the decision to buy insurance, or not. The expected premium is given by:

$$E[\Pi] = d(\mu_1, \pi_1)p_1\pi_1 + d(\mu_2, \pi_2)p_2\pi_2. \tag{3}$$

$E[\Pi]$ corresponds to the final column of the fourth row in the tables in Section 2. Since individuals who do not buy insurance pay premium zero, we can also write $E[\Pi] = E[Q\Pi]$.

The insurance claim actually made by an individual chosen at random is QL. The expected insurance claim — equivalent to the ‘loss coverage’ heuristically defined in Section 2 — is given by:

$$\text{Loss coverage: } E[QL] = d(\mu_1, \pi_1)p_1\mu_1 + d(\mu_2, \pi_2)p_2\mu_2, \tag{4}$$

where we assume no moral hazard, i.e. purchase of insurance has no bearing on the risk. Loss coverage can also be thought of as *risk-weighted* insurance demand. Note that we do not normalize loss coverage, i.e. define it to be $E[QL]/L$, because L is a random variable and not a deterministic amount of loss.

A formal probabilistic framework for the above set-up is provided in Appendix A.

3.2 Demand for insurance

In the previous section, we have introduced the concept of proportional demand for insurance, $d(\mu_i, \pi_i)$, when a premium π_i is charged for risk-group with true risk μ_i (in fact, $d(\mu_i, \pi_i)$ was defined to be a probability). In this section, we specify a functional form for $d(\mu_i, \pi_i)$ and its relevant properties.

This form of demand function differs from others found in the literature, in that we assume that everyone either does or does not buy insurance which fully covers their potential loss. Other approaches would be that individuals decide what level of cover to buy (possibly zero) or what deductible to choose. This would be a useful area for future research.

De Jong & Ferris (2006) suggested axioms for an insurance demand function, adapted below using our notation:

(a) $d(\mu_i, \pi_i)$ is a decreasing function of premium π_i for all risk-groups i;
(b) $d(\mu_1, \pi) < d(\mu_2, \pi)$, i.e. at a given premium π, the proportional demand is greater for the higher risk-group;
(c) $d(\mu_i, \pi_i)$ is a decreasing function of the premium loading π_i/μ_i; and
(d) for our model, where all losses are of unit size, we need to add $d(\mu_i, \pi_i) \leq 1$, i.e. the highest possible demand is when all members of the risk-group buy insurance.

The case of actuarially fair premiums, $\pi_i = \mu_i$, is of special interest. We define $\tau_i = d(\mu_i, \mu_i)$ to be the "fair-premium demand" for insurance for risk-group i, which can be regarded as the proportion of risk-group i who buy insurance at an actuarially fair premium

De Jong & Ferris (2006) suggested a "flexible but practical" exponential-power demand function, and this approach was also followed by Thomas (2008, 2009). However the exponential-power function, whilst very flexible, is also rather intractable. In the present paper, we use a more tractable function which satisfies the axioms above and for which the price elasticity of demand in risk-group i is a positive constant λ_i, i.e.:

$$-\frac{\pi_i}{d(\mu_i, \pi_i)} \frac{\partial d(\mu_i, \pi_i)}{\partial \pi_i} = \lambda_i. \tag{5}$$

Solving Equation 5 leads to the following functional form for demand:

$$d(\mu_i, \pi_i) = \tau_i \left(\frac{\pi_i}{\mu_i} \right)^{-\lambda_i}. \tag{6}$$

This equation specifies demand as a function of the "premium loading" (π_i/μ_i). When the premium loading is high (insurance is expensive), demand is low, and vice versa. The λ_i

parameter controls the shape of the demand curve. The "iso-elastic" terminology reflects that the price elasticity of demand is the same everywhere along the demand curve.

Clearly, iso-elastic demand functions satisfy axioms (a) and (c) of De Jong & Ferris (2006). Axioms (b) and (d) appear superficially to require conditions on the fair-premium demands τ_1 and τ_2. However, if we define *fair-premium demand-shares* α_1 and α_2 as:

$$\text{Fair-premium demand-share: } \alpha_i = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}, \quad i = 1, 2 \tag{7}$$

then it turns out that the properties of most interest in the model depend just on α_1 (clearly, $\alpha_2 = 1 - \alpha_1$). It will suffice to analyse the model for the full range of fair-premium demand-shares $0 \leq \alpha_1 \leq 1$ without specifying the p_i and τ_i. It is enough to know that for every possible α_1 there must exist some combination of p_i and τ_i which satisfies the axioms (b) and (d) above.

4. Equilibrium

In the model in Section 3, an insurance market equilibrium is reached when the premiums charged (π_1, π_2) ensure that the expected profit, $f(\pi_1, \pi_2) = 0$, where:

$$\begin{aligned} f(\pi_1, \pi_2) &= E[\Pi] - E[QL] & (8) \\ &= d(\mu_1, \pi_1)(\pi_1 - \mu_1)p_1 + d(\mu_2, \pi_2)(\pi_2 - \mu_2)p_2. & (9) \end{aligned}$$

4.1 Risk-differentiated Premiums

The profit equation $f(\pi_1, \pi_2) = 0$ is obviously satisfied if $(\pi_1, \pi_2) = (\mu_1, \mu_2)$. Setting premiums equal to the respective risks results in an expected profit of zero for each risk group and also in aggregate. We shall refer to this case as *risk-differentiated premiums.*

Following the notation introduced in Section 3, the expected insurance coverage in this case is given by:

$$E[Q] = \tau_1 p_1 + \tau_2 p_2. \tag{10}$$

Also, (π_1, π_2) being equal to (μ_1, μ_2), the expected premium and expected claim are equal and given by:

$$E[\Pi] = E[QL] = \tau_1 p_1 \mu_1 + \tau_2 p_2 \mu_2. \tag{11}$$

4.2 Equilibrium Pooled Premium

Next we consider the case of an *equilibrium pooled premium.* This is where risk classification is banned, so that insurers have to charge the same premium π_0 for both risk-groups, i.e. $\pi_1 = \pi_2 = \pi_0$, leading at equilibrium to $f(\pi_0, \pi_0) = 0$. For convenience, we omit one argument for all bivariate functions if both arguments are equal, e.g. we write $f(\pi)$ for $f(\pi, \pi)$.

Equation 8 leads to the following relationship for the equilibrium pooled premium π_0:

$$\pi_0 = \frac{E[QL]}{E[Q]}. \tag{12}$$

The existence of a solution for $f(\pi) = 0$ within the interval $(\mu_1, \mu_2]$ is obvious, because $f(\pi)$ is a continuous function with $f(\mu_1) < 0$ and $f(\mu_2) \geq 0$. However, uniqueness of the solution is not guaranteed.

The equilibrium pooled premium π_0 depends on the demand elasticities λ_1 and λ_2. If $\lambda_1 = \lambda_2$ then π_0 should satisfy some constraints which are easily deduced from economic considerations, and in Section 4.2.1 we show that this is so. However, it is more realistic to expect that $\lambda_1 < \lambda_2$, because of the income effect on demand (i.e. for high risks the cost of insurance represents a larger part of an individual's total budget constraint, so their elasticity of demand for insurance is likely to be higher). We consider unequal demand elasticities in Section 4.2.2, and find that the key to the properties of π_0 lies in a linear relationship between λ_1 and λ_2 for fixed values of π_0.

4.2.1 *Equal Demand Elasticities*

Assuming an iso-elastic demand function with $\lambda_1 = \lambda_2 = \lambda$, Equation 12 provides a unique solution:

$$\pi_0 = \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1^{\lambda} + \alpha_2 \mu_2^{\lambda}}. \tag{13}$$

This can be written as a weighted average of the true risks μ_1 and μ_2:

$$\pi_0 = v\mu_1 + (1-v)\mu_2, \quad \text{where} \quad v = \frac{\alpha_1}{\alpha_1 + \alpha_2 \left(\frac{\mu_2}{\mu_1}\right)^{\lambda}}. \tag{14}$$

Note that π_0 does not depend directly on the individual values of the population fractions (p_1, p_2) and fair-premium demands (τ_1, τ_2), but only indirectly on these parameters through the demand-shares (α_1, α_2). In other words, populations with the same true risks (μ_1, μ_2) and demand-shares (α_1, α_2) have the same equilibrium premium, even if the underlying (p_1, p_2) and (τ_1, τ_2) are different.

Figure 1 plots the pooled equilibrium premium against demand elasticity, λ, for two different population structures with the same true risks $(\mu_1, \mu_2) = (0.01, 0.04)$ but different fair-premium demand-shares (α_1, α_2).

The following observations can be derived from Equations 13 and 14, and are illustrated by Figure 1:

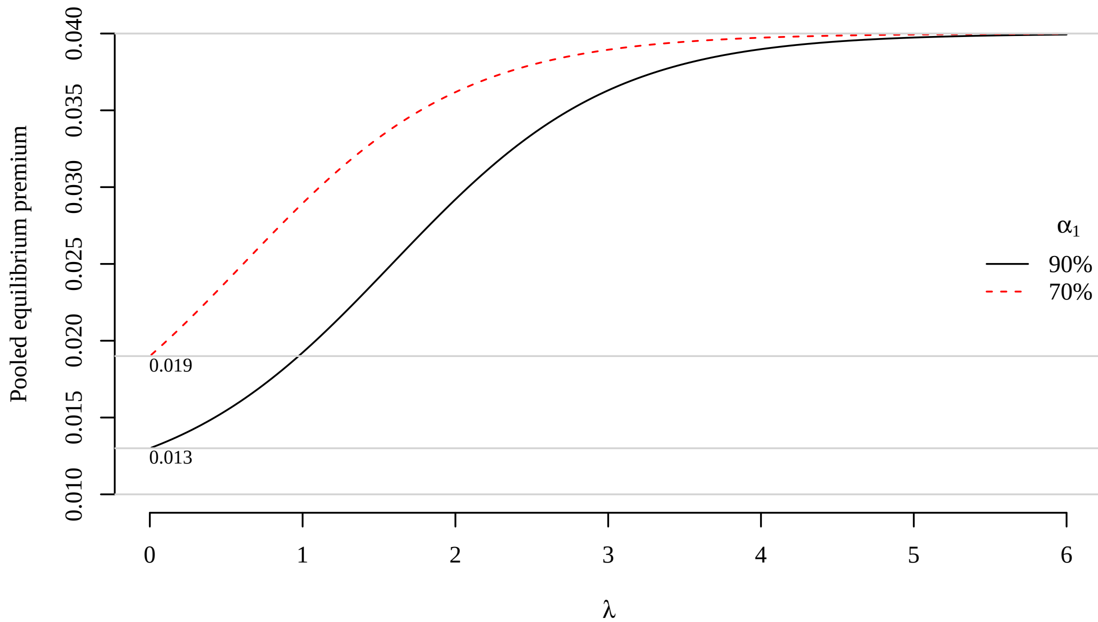

Figure 1: Pooled equilibrium premium as a function of λ for two populations with the same $(\mu_1, \mu_2) = (0.01, 0.04)$ but different values of α_1.

(a)

$$\lim_{\lambda \to 0} \pi_0 = \alpha_1 \mu_1 + \alpha_2 \mu_2. \tag{15}$$

Intuitively, if demand is inelastic, changing the premium makes no difference, and so the equilibrium premium will be the same as the expected claim per policy if risk-differentiated premiums were charged. In Figure 1, this is 0.013 and 0.019 for fair-premium demand-shares of $\alpha_1 = 0.9$ and $\alpha_1 = 0.7$ respectively.

(b)

$$\pi_0 \text{ is an increasing function of } \lambda. \tag{16}$$

Intuitively, an increase in demand elasticity means that at any premium between μ_1 and μ_2, there will be less demand than before from low risks and more demand than before from high risks; the premium for which profits on low risks exactly balance losses on high risks will therefore be higher. In Figure 1, both curves slope upwards. In Equation 14, increasing λ reduces the weight w on low-risk, resulting in an increase in the equilibrium premium π_0.

(c)

$$\lim_{\lambda \to \infty} \pi_0 = \mu_2. \tag{17}$$

Intuitively, if demand elasticity is very high, demand from the low risk-group falls to zero for any premium above their true risk μ_1. The only remaining insureds are then all high risks, so the equilibrium premium must move to $\pi_0 = \mu_2$. In Figure 1, both curves converge to $\mu_2 = 0.04$ as λ increases.

(d)

$$\pi_0 \text{ is a decreasing function of } \alpha_1. \tag{18}$$

Intuitively, if the fair-premium demand-share α_1 of the lower risk-group increases, we would expect the equilibrium premium to fall. In Figure 1, the curve for $\alpha_1 = 0.9$ lies below the curve for $\alpha_1 = 0.7$.

4.2.2 *Different Demand Elasticities*

Where demand elasticities are not the same, any equilibrium premium π_0 can be consistent with many different elasticity pairs (λ_1, λ_2). For iso-elastic demand, the elasticity pairs consistent with any particular equilibrium premium turn out to be linearly related. This is illustrated in Figure 2.

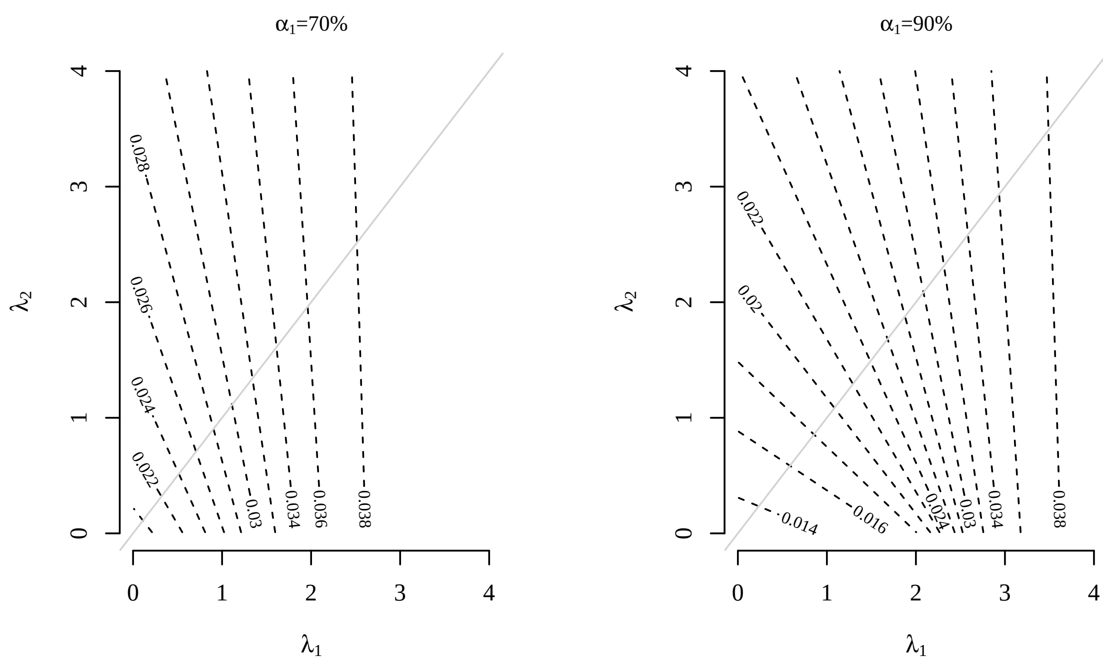

Figure 2: Equilibrium premium as a function of (λ_1, λ_2) for $\alpha_1 = 70\%$ and $\alpha_1 = 90\%$, when $(\mu_1, \mu_2) = (0.01, 0.04)$.

Figure 2 shows contour plots of constant pooled equilibrium premiums. The left-hand panel shows a population with fair-premium demand-share $\alpha_1 = 70\%$, and the right-hand panel shows a population with $\alpha_1 = 90\%$. Each straight dashed line represents all the pairs of λ_1 and λ_2 values consistent with one particular equilibrium premium. As expected, each straight dashed line has negative slope: if one elasticity parameter rises, the other elasticity parameter must fall, if the equilbrium premium is to stay the same. Note that the 45-degree diagonal ($\lambda_1 = \lambda_2$) in each panel corresponds to the case of equal demand elasticities represented by the respective curves in Figure 1.

The patterns in the plots in Figure 2 can be explained conveniently if Equation 9, with the iso-elastic demand function, is re-arranged as follows:

$$\lambda_1 \log\left(\frac{\pi_0}{\mu_1}\right) + \lambda_2 \log\left(\frac{\mu_2}{\pi_0}\right) = \log\left(\frac{\alpha_1(\pi_0 - \mu_1)}{\alpha_2(\mu_2 - \pi_0)}\right). \tag{19}$$

We observe the following:

(a) Equation 19 expresses a linear relationship between λ_1 and λ_2, for a fixed value π_0 of the equilibrium pooled premium. This produces the linear patterns in the contour plots of Figure 2.

(b)
$$\lim_{(\lambda_1,\lambda_2)\to(0,0)} \pi_0 = \alpha_1\mu_1 + \alpha_2\mu_2. \tag{20}$$
This follows directly from Equation 19. Intutively, if demand is inelastic, the equilibrium pooled premium will be close to the expected claim under fair premiums.

(c)
$$\pi_0 \geq \alpha_1\mu_1 + \alpha_2\mu_2. \tag{21}$$
This follows from the fact that $\mu_1 \leq \pi_0 \leq \mu_2$, $\lambda_1 \geq 0$, $\lambda_2 \geq 0$ and the relationship in Equation 19. Intuitively, the equilibrium pooled premium is never smaller than the expected claim under fair premiums.

(d)
$$\lim_{(\lambda_1,\lambda_2)\to(\infty,\lambda_2)} \pi_0 = \mu_2, \tag{22}$$
which again follows from Equation 19. Intuitively, high demand elasticities lead to an equilibrium where only high risks purchase insurance.

(e) Given π_0:
$$\frac{\log(\pi_0/\mu_1)}{\log(\mu_2/\pi_0)} \text{ is an increasing function of } \pi_0, \tag{23}$$
i.e., the (absolute value of the) slope of the line, in Equation 19 increases with π_0. Intuitively, a higher equilibrium premium π_0 is consistent with higher sensitivity to λ_2 and lower sensitivity to λ_1. In the limit, as $\pi_0 \to \mu_2$, the straight line in Equation 19 becomes perpendicular to the λ_1-axis, as can be seen from Figure 2.

(f) Given π_0:

$$\lim_{\lambda_1 \to 0} \lambda_2 = \frac{\log\left(\frac{\alpha_1(\pi_0-\mu_1)}{\alpha_2(\mu_2-\pi_0)}\right)}{\log\left(\frac{\mu_2}{\pi_0}\right)} \text{ is an increasing function of } \pi_0, \tag{24}$$

i.e., the intercept on the λ_2-axis in the plots of Figure 2 increases with π_0. Intuitively, if the low-risk group is insensitive to premiums, a higher equilibrium premium π_0 is consistent with higher demand elasticity λ_2 for of high risks, because this increases the demand from that group at any premium $\pi_0 < \mu_2$.

(g) Given π_0, changing the fair-premium demand-share α_1 results in parallel shifts of the straight lines given in Equation 19, as the slopes remain unchanged while the intercepts are adjusted accordingly. In Figure 2, changing α_1 from 70% to 90% has the effect of translating the contours towards the top-right corner. It also confirms that increasing the fair-premium demand-share α_1 results in a decrease in equilibrium premium, because the impact of the low risk-group increases.

4.3 Multiple Equilibria

In Section 4.2, we noted that the existence of an equilibrium pooled premium in our model is obvious, but its uniqueness is not. That multiple equilibria can arise was demonstrated by Thomas (2009) using the exponential power demand function proposed by De Jong & Ferris (2006). Thus, for any choice of demand function, we must determine whether or not multiple equilibria can arise, and if they can, whether or not they are material. We can show that multiple equilibria can arise under an iso-elastic demand function, but only under two conditions that are practically ruled out by economic considerations, so that that multiple equilibria are unlikely to be troublesome in any practical application. These conditions are as follows.

- Demand elasticity λ_1 for the low risk-group is substantially higher than demand elasticity λ_2 for the high risk-group. This is the opposite of what we would expect, because of the income effect on demand mentioned in Section 4.2.

- The low risk-group has fair-premium demand-share within a very narrow range of very high values. Loosely speaking, this means that the high risk-group must be very small relative to the total population.

However, in a competitive market only the smallest equilibrium premium should matter (see Hoy & Polborn (2000)), and the proof of the above conditions is lengthy, so we omit it.

5. Adverse Selection

Evidence of adverse selection is typically demonstrated in the economics literature as positive correlation (or equivalently, covariance) of coverage and losses (e.g. for a survey see Cohen & Siegelman (2010)). Using the notations developed in Section 3, this can be quantified by the covariance between the random variables Q and L, i.e. $E[QL] - E[Q]E[L]$. We prefer to use the ratio rather than the difference, so our definition is:

$$\text{Adverse selection: } S(\pi_1, \pi_2) = \frac{E[QL]}{E[Q]E[L]}. \tag{25}$$

Using Equations 7, 10 and 11, adverse selection under risk-differentiated premiums is:

$$S(\mu_1, \mu_2) = \frac{\tau_1 p_1 \mu_1 + \tau_2 p_2 \mu_2}{\tau_1 p_1 + \tau_2 p_2} \times \frac{1}{E[L]} = \frac{\alpha_1 \mu_1 + \alpha_2 \mu_2}{E[L]}. \tag{26}$$

In the particular case of pooled equilibrium premium, π_0, by Equation 12, we have:

$$S(\pi_0) = \frac{\pi_0}{E[L]}. \tag{27}$$

However, since by Equation 21, $\pi_0 \geq \alpha_1 \mu_1 + \alpha_2 \mu_2$, we have:

$$S(\pi_0) \geq S(\mu_1, \mu_2). \tag{28}$$

In other words, adverse selection is always higher under pooling than under risk-differentiated premiums. Therefore it cannot serve as a measure of better outcomes for society as a whole (Table 2 in the motivating examples in Section 2) or worse outcomes for society as a whole (Table 3 in the motivating examples in Section 2). This leads us to the concept of loss coverage ratio discussed in the next section.

6. Loss Coverage

The motivating examples in Section 2 suggested loss coverage — heuristically characterised as the proportion of the population's losses compensated by insurance — as a measure of the social efficacy of insurance. This can be formally quantified in our model by the expected insurance claim, $E[QL]$, defined in Section 3 as:

$$\text{Loss coverage: } LC(\pi_1, \pi_2) = E[QL]. \tag{29}$$

To compare the relative merits of different risk classification regimes, we need to define a reference level of loss coverage. We use the level under risk-differentiated premiums, and so define the *loss coverage ratio*, as follows:

$$\text{Loss coverage ratio: } C = \frac{LC(\pi_1, \pi_2)}{LC(\mu_1, \mu_2)}. \tag{30}$$

In the following sections we consider the loss coverage ratio when the equilibrium pooled premium is charged, i.e. $\pi_1 = \pi_2 = \pi_0$. We are particularly interested in establishing when the loss coverage ratio may exceed unity, The case of unequal demand elasticities is considered in Section 6.2. As with the equilibrium pooled premium π_0 itself, the properties of the loss coverage ratio are explored by finding relationships between λ_1 and λ_2 for fixed values of π_0. In this case, the relationships are log-linear rather than linear. It is then possible to determine values of λ_1 and λ_2 for which the loss coverage ratio exceeds unity. Arguably, this region includes plausible values of λ_1 and λ_2.

6.1 Equal Demand Elasticities

As for the equilibrium pooled premium, we first analyse the properties of the loss coverage ratio in the special case of equal demand elasticities, i.e. $\lambda_1 = \lambda_2 = \lambda$. Using the iso-elastic demand function in Equation 4 leads to:

$$C(\lambda) = \frac{1}{\pi_0^\lambda} \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1 + \alpha_2 \mu_2}, \tag{31}$$

where π_0 is the pooled equilibrium premium given in Equation 13. The above can also be conveniently re-expressed as:

$$C(\lambda) = \frac{\left[w\beta^{1-\lambda} + (1-w)\right]^\lambda \left[w + (1-w)\beta^\lambda\right]^{1-\lambda}}{\beta^{\lambda(1-\lambda)}}, \quad \text{where} \tag{32}$$

$$w = \frac{\alpha_1 \mu_1}{\alpha_1 \mu_1 + \alpha_2 \mu_2}, \quad \text{and} \tag{33}$$

$$\beta = \frac{\mu_2}{\mu_1} > 1. \tag{34}$$

Figure 3 shows loss coverage ratio for four population structures. Both plots in Figure 3 show the same example, with the right-hand plot zooming over the range $0 < \lambda < 1$. We make the following observations:

(a)

$$\lim_{\lambda \to 0} C(\lambda) = 1. \tag{35}$$

This follows directly from Equation 31. Intuitively, if demand is inelastic then pooling must give the same loss coverage as fair premiums.

(b)

$$\lim_{\lambda \to \infty} C(\lambda) = 1 - w = \frac{\alpha_2 \mu_2}{\alpha_1 \mu_1 + \alpha_2 \mu_2}. \tag{36}$$

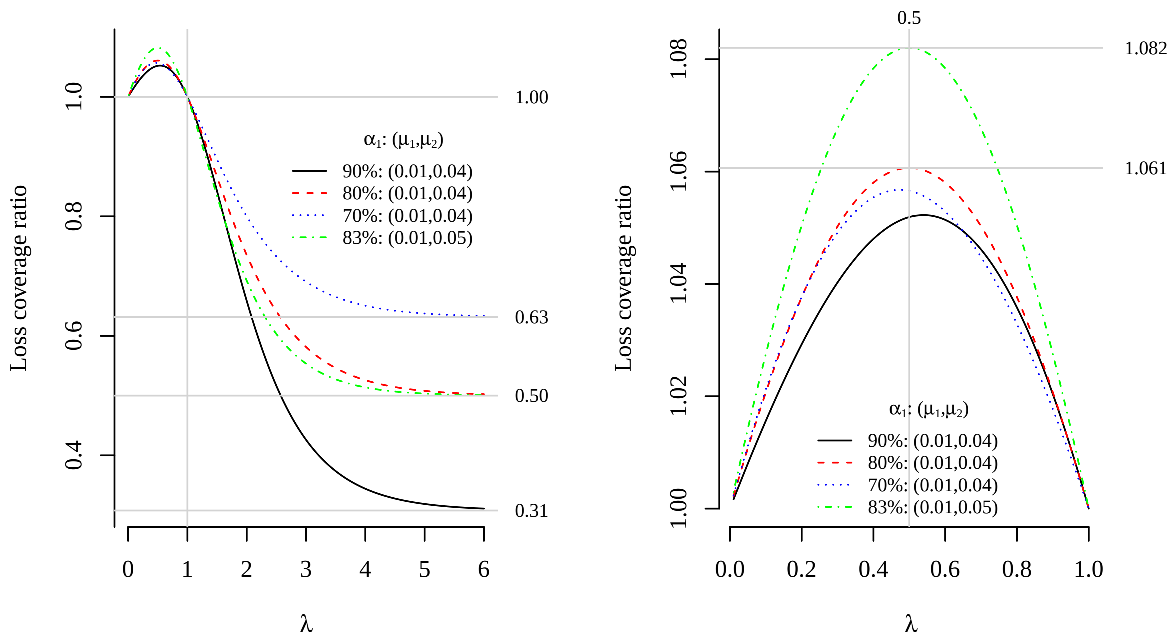

Figure 3: Loss coverage ratio as a function of λ for four population structures.

This follows from Equation 32, by taking the denominator, $\beta^{\lambda(1-\lambda)}$, inside the second term in the numerator and noting that $\beta > 1$. Recall that for highly elastic demand, equilibrium is achieved when only high risks buy insurance at the equilibrium premium $\pi_0 = \mu_2$, which explains the above result. The left-hand plot of Figure 3 shows that the limiting loss coverage ratio increases with increasing weight $(1-w)$ of high risks' contribution to loss coverage under fair premiums. When the population structure and relative risks are such that the high risks and low risks each contribute equal weight to the loss coverage under fair premiums, the limiting loss coverage ratio is 0.5.

(c) For $\lambda > 0$,

$$\lambda \lesseqqgtr 1 \Rightarrow C(\lambda) \gtreqqless 1. \tag{37}$$

The proof of this result is outlined in Appendix B. The result implies that pooling produces higher loss coverage than fair premiums if demand elasticity is less than 1.

(d)

$$\max_{w,\lambda} C = \frac{1}{2}\left(\sqrt[4]{\frac{\mu_2}{\mu_1}} + \sqrt[4]{\frac{\mu_1}{\mu_2}}\right) = \frac{1}{2}\left(\sqrt[4]{\beta} + \frac{1}{\sqrt[4]{\beta}}\right). \tag{38}$$

The proof of this result is also provided in Appendix B. As can be seen from the right-hand plot of Figure 3, for a given value of relative risk, β, the loss coverage ratio attains its maximum when $\lambda = 0.5$ and $w = 0.5$. Moreover, the maximum loss coverage ratio increases with increasing relative risk. This implies that a pooled premium might be highly beneficial in the presence of a small group with very high risk exposure. Hoy (2006) obtained a similar result based on social welfare, so there are at least two different normative justifications for pooling very different insurance risks.

6.2 Different Demand Elasticities

In the general case, where the demand elasticities are allowed to be different, the loss coverage ratio is given by:

$$C(\lambda_1, \lambda_2) = \frac{\alpha_1\mu_1 \left(\frac{\pi_0}{\mu_1}\right)^{-\lambda_1} + \alpha_2\mu_2 \left(\frac{\pi_0}{\mu_2}\right)^{-\lambda_2}}{\alpha_1\mu_1 + \alpha_2\mu_2}, \tag{39}$$

where π_0 is an equilibrium premium satisfying Equation 19. Using the relationship between λ_1, λ_2 and π_0 in Equation 19, we can express loss coverage ratio in Equation 39 in either of these two alternative forms:

$$\log C = -\lambda_1 \log\left(\frac{\pi_0}{\mu_1}\right) + \log k_1, \quad \text{where} \quad k_1 = \frac{\alpha_1(\mu_2 - \mu_1)\pi_0}{(\alpha_1\mu_1 + \alpha_2\mu_2)(\mu_2 - \pi_0)}, \tag{40}$$

$$\log C = +\lambda_2 \log\left(\frac{\mu_2}{\pi_0}\right) + \log k_2, \quad \text{where} \quad k_2 = \frac{\alpha_2(\mu_2 - \mu_1)\pi_0}{(\alpha_1\mu_1 + \alpha_2\mu_2)(\pi_0 - \mu_1)}. \tag{41}$$

Equations 40 and 41 show that given an equilibrium premium, π_0, the loss coverage ratio can be expressed as a log-linear function of either λ_1 or λ_2. Figure 4 shows the graphical representations of Equations 40 and 41, for different values of α_1 when $(\mu_1, \mu_2) = (0.01, 0.04)$. We make the following observations:

(a) Given an equilibrium premium, π_0, the loss coverage ratio is an increasing function of λ_2 and, consequently, a decreasing function of λ_1. Recall that Equation 19 implies that, in order to keep the equilibrium premium constant, increasing λ_2 would require decreasing λ_1. But both increasing λ_2 or decreasing λ_1 have the same effect of increasing demand from the respective risk-groups, leading to an overall increase in the loss coverage ratio.

(b) As a consequence, given an equilibrium pooled premium, π_0, the loss coverage ratio is:
   (1) maximum when $\lambda_1 = 0$ and takes the value k_1 ; and
   (2) minimum when $\lambda_2 = 0$ and takes the value k_2.

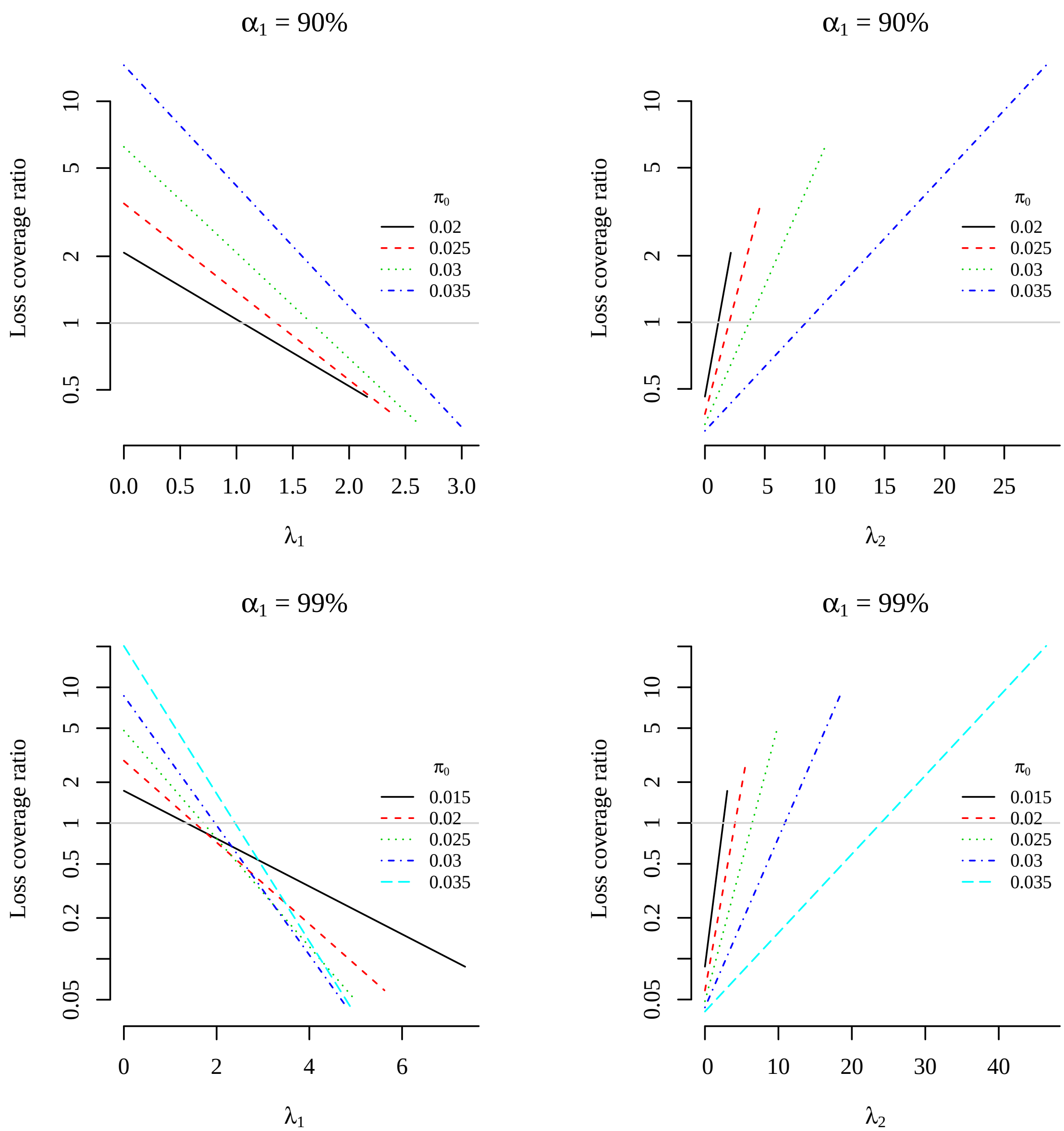

Figure 4: Loss coverage ratio (log scale) as functions of λ_1 and λ_2 for different values of equilibrium premiums, when $(\mu_1, \mu_2) = (0.01, 0.04)$ and $\alpha_1 = 90\%$ and 99%.

(c) For a given value of λ_2, the loss coverage ratio is a decreasing function of λ_1. This can be obtained as follows:

$$\frac{d}{d\lambda_1}\log C = \frac{\partial}{\partial\lambda_1}\log C + \left(\frac{\partial}{\partial\pi_0}\log C\right)\left(\frac{d\pi_0}{d\lambda_1}\right) < 0, \quad \text{since} \tag{42}$$

$$\frac{\partial}{\partial\lambda_1}\log C = -\log\left(\frac{\pi_0}{\mu_1}\right) < 0, \quad \text{by Equation 40,} \tag{43}$$

$$\frac{\partial}{\partial\pi_0}\log C = -\frac{\lambda_2}{\pi_0} - \frac{\mu_1}{\pi_0(\pi_0-\mu_1)} < 0, \quad \text{by Equation 41,} \tag{44}$$

$$\frac{d\pi_0}{d\lambda_1} > 0, \quad \text{provided the equilibrium premium is unique.} \tag{45}$$

All else being fixed, including λ_2, the equilibrium pooled premium can only increase if λ_1 increases. Higher demand elasticity and higher equilibrium pooled premium both imply a fall in low-risk demand. A higher equilibrium pooled premium also reduces high-risk demand. Since demand from both risk-groups falls, the loss coverage ratio falls.

(d) However for a given value of λ_1, there is no monotonic relationship between the loss coverage ratio and the equilibrium pooled premium, π_0, as Equation 40 gives:

$$\frac{\partial}{\partial\pi_0}\log C = 0 \Rightarrow \pi_0 = \frac{\lambda_1 - 1}{\lambda_1}\mu_2, \tag{46}$$

with positive second derivative, indicating a possible minimum for π_0 in the range $\mu_1 < \pi_0 < \mu_2$. So a non-monotonic relationship between loss coverage ratio and λ_2 is possible. This is illustrated in the left panel of Figure 4, where the crossover of the lines for different equilibrium pooled premiums implies a non-monotonic ordering of premiums by loss coverage ratio for some values of λ_1. This effect arises because for high risks, an increase in premium and increase in elasticity have opposite effects on demand. The sum of these effects plus the fall in low-risk demand determine the change in the loss coverage ratio, which can either rise or fall.

(e) Focussing on demand elasticities less than 1, Figure 5 demarcates the regions where the loss coverage ratio is greater than or less than 1. We make the following observations:

(1) For $0 < \lambda_1 < \lambda_2 < 1$, loss coverage ratio exceeds 1. Given $\lambda_1 < \lambda_2$, let π_0^* be the resulting equilibrium premium. Then by Equation 19, there exists a common demand elasticity, λ^*, for both risk-groups, where $\lambda_1 < \lambda^* < \lambda_2$, which leads to the same equilibrium premium, π_0^*. However we know that, if the demand elasticities are equal and less than 1, then the loss coverage ratio exceeds 1, i.e. $C(\lambda^*, \lambda^*) > 1$. But, as $\lambda_2 > \lambda^*$, by Equation 41, $C(\lambda_1, \lambda_2) > C(\lambda^*, \lambda^*) > 1$.

(2) For $0 < \lambda_2 < \lambda_1 < 1$, the curve showing loss coverage ratio of 1 becomes increasingly more convex up to certain limit, as β increases. In other words, as the

relative risk increases, more combinations of (λ_1, λ_2) produce loss coverage ratio greater than 1.

As discussed below in Section 7, there is some empirical evidence that insurance demand elasticities are typically less than 1 in many markets. Also, any gradient in demand elasticity from low to high risks might be expected to be positive, because of the income effect on demand. That is, for high risks the cost of insurance represents a larger part of consumers' total budget constraint, so their elasticity of demand for insurance might be larger. Hence Figure 5 suggests that for realistic levels of demand elasticities, loss coverage ratio may typically exceed 1. This result gets stronger with increasing relative risks (the curve above which the loss coverage ratio is greater than 1 becomes more convex).

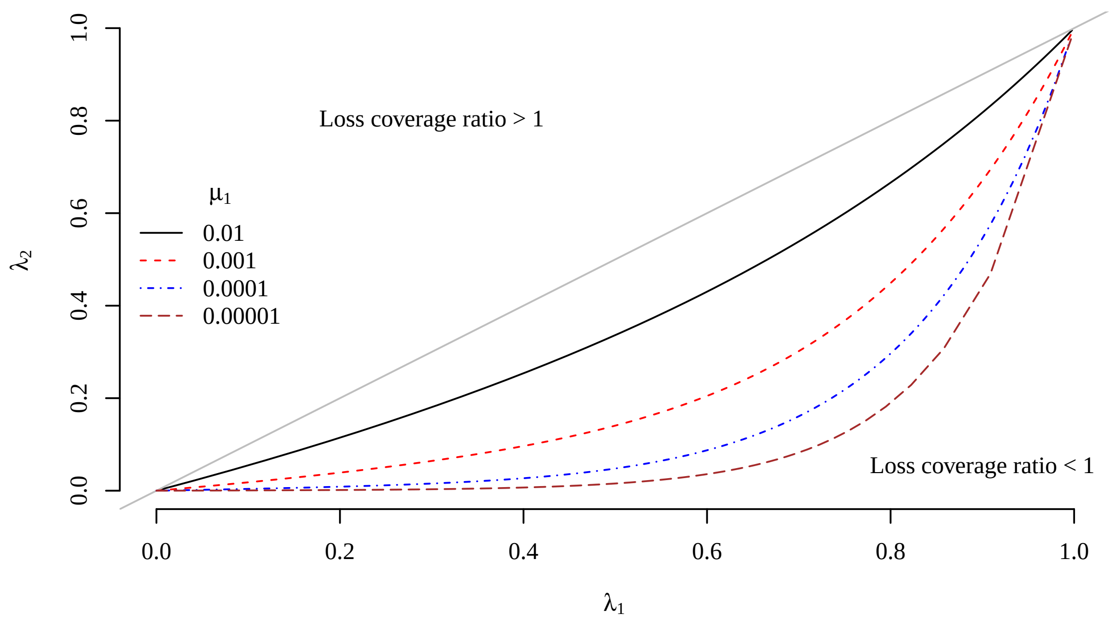

Figure 5: Curves demarcating the regions where loss coverage ratio is greater than and less than 1 for different values of μ_1 when $\alpha_1 = 0.9$ and $\mu_2 = 0.04$.

7. Summary and Conclusions

The results in preceding sections can be summarised and interpreted as follows.

Adverse selection — at an equilibrium where the insurer just breaks even — is always higher under pooling than under risk-differentiated premiums. On the other hand, loss coverage can be be higher or lower under pooling than under risk-differentiated premiums. Loss coverage — the expected losses compensated by insurance — is higher under pooling if the shift in coverage towards higher risks more than compensates for the fall in number of risks insured.

For iso-elastic demand with equal demand elasticities in high and low risk-groups, $\lambda_1 = \lambda_2 = \lambda$, the equilibrium pooled premium (EPP) and loss coverage ratio (LCR) can be characterised as follows.

(a) Under pooling, EPP increases monotonically with λ, tending to an upper limit where the only remaining insureds are high risks.
(b) Under pooling, if $\lambda < 1$ then LCR > 1.
(c) As λ increases from zero, LCR increases to a maximum at around $\lambda = 0.5$; then decreases to 1 when $\lambda = 1$; and then flattens out at a lower limit for high values of λ, where the only remaining insureds are high risks.
(d) The maximum value of LCR, attained for λ about 0.5, depends on the relative risk, $\beta = \mu_2/\mu_1$. A higher β gives a higher maximum value of LCR.

For iso-elastic demand with different demand elasticities λ_1 and λ_2 in high and low risk-groups, respectively, EPP and LCR can be characterised as follows:

(a) At a given EPP, there is a linear relationship between all the feasible pairs of demand elasticities (λ_1, λ_2).
(b) Given λ_2, LCR is a decreasing function of λ_1.
(c) On the other hand, given λ_1, LCR is not necessarily a monotonic function of λ_2.
(d) For $\lambda_1 < \lambda_2 < 1$, LCR is always greater than 1.
(e) For other values of λ_1 and λ_2, LCR > 1 if λ_1 is 'sufficiently low' compared with λ_2. The value of λ_1 which is 'sufficiently low' may be greater or less than λ_2. We did not find any general conditions on $(\lambda_1\ ,\ \lambda_2)$ that guaranteed LCR > 1.
(f) As relative risk β increases, more combinations of (λ_1, λ_2) result in LCR > 1.
(g) Multiple equilibria are theoretically possible, but they arise only for extreme population structures combined with implausible elasticity parameters, where *both* (i) the fair-premium demand-share α_1 of the low risk-group is in a narrow range of high values *and* (ii) λ_1 is substantially higher than λ_2. Therefore multiple equilibria are not likely to be a practical concern.

We suggest loss coverage — the expected losses compensated by insurance for the whole population — as a reasonable metric for the social efficacy of insurance. If this is accepted, and if our iso-elastic model of insurance demand is reasonable, then pooling will be beneficial:

(a) in the equal elasticities case, whenever $\lambda < 1$; and
(b) in the different elasticities case, if λ_1 is sufficiently low, compared with λ_2.

There is some empirical evidence that insurance demand elasticities are typically less than 1 in many markets. We defined demand elasticity as a positive constant in Equation 5, but the estimates in empirical papers are generally given with the negative sign, and so we quote them in that form. For example, for yearly renewable term insurance in the US, an estimate of -0.4 to -0.5 has been reported (Pauly et al, 2003). A questionnaire survey about life insurance purchasing decisions produced an estimate of -0.66 (Viswanathan et al, 2007). For private health insurance in the US, several studies estimate demand elasticities in the range of 0 to -0.2 (Chernew et al., 1997; Blumberg et al., 2001; Buchmueller & Ohri, 2006). For private health insurance in Australia, Butler (1999) estimates demand elasticities in the range -0.36 to -0.50. These magnitudes are consistent with the possibility that loss coverage might sometimes be increased by restricting risk classification.

Our model considers only two possibilities for risk classification, fully risk-differentiated premiums or complete pooling. In practice, it is common to see partial restrictions on risk classification, where particular risk factors such as gender or genetic test results or family history are banned. Our model does not explicitly consider such scenarios. However, we note that in our model, loss coverage is maximised when there is an intermediate level of adverse selection, not too low and not too high. It is possible that in some markets, complete pooling generates too much adverse selection; but partial restrictions on risk classification generate an intermediate level of adverse selection, and hence higher loss coverage than either pooling or fully risk-differentiated premiums.

Thus from a public policy perspective, the concept of loss coverage offers a possible rationale for some degree of restriction on risk classification. Loss coverage also provides a metric for assessing, in particular cases, whether the degree of restriction produces a good or bad result for the population as a whole. Insurers typically take a different view, arguing against any and all restrictions on risk classification. However, note that from the insurance industry's perspective, maximising loss coverage is equivalent to maximising premium income. Our model assumes that insurers make zero profits in equilibrium under all risk classification schemes, but in practice insurers hope to earn profits. If these profits are proportional to premiums, restrictions on risk classification which maximise loss coverage could be advantageous to the insurance industry. In other words, the concept of loss coverage suggests that adverse selection is not always a bad thing, even for insurers.

We recognize that loss coverage is one among many possible measures of the benefit that insurance markets may bring to society. An alternative perspective (suggested by a referee) is that actuarially fair pricing reveals preferences, and therefore when we move to pooling, more value is lost by those who cease to buy insurance than is gained by those who now choose to buy insurance. Each measure will have its merits and demerits.

Future work to extend and apply these results could include: investigating equilibrium premium, adverse selection and loss coverage for other insurance demand functions; investigation of the effects of partial restrictions on risk classification; and empirical investigations of insurance demand.

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Appendices

A. Probabilistic Framework of the Model

Consider a sample space $\Omega = \{1, 2, \ldots, N\}$ of N risks. Let $A_1, A_2, \ldots, A_n$ denote a partition of Ω (i.e. $A_i \cap A_j = \varnothing$ for $i \neq j$ and $\cup_{i=1}^n A_i = \Omega$), where A_i represents the i-th risk-group. Define the counting probability measure: $P[\{\omega\}] = 1/N$ for $\omega \in \Omega$, so that $P[A_i] = \#(A_i)/N$, which will be denoted by p_i for all $i = 1, 2, \ldots, n$. Let X be any indicator random variable on Ω, taking the values 0 or 1. Then:

$$E[X] = \sum_{i=1}^{n} P[\,A_i\,]\,E[X \mid A_i\,] \tag{47}$$

$$= \sum_{i=1}^{n} p_i\,P[X = 1 \mid A_i]. \tag{48}$$

We define the following indicator random variables:

$$\begin{aligned} L(\omega) &= I[\,\text{individual } \omega \text{ incurs a loss}\,] \\ Q(\omega) &= I[\,\text{individual } \omega \text{ buys insurance}\,]. \end{aligned}$$

We assume that both L and Q, restricted to the risk group A_i, are independent Bernoulli random variables with parameters as follows:

$$P[L = 1 \mid A_i] = \mu_i \tag{49}$$

$$P[Q = 1 \mid A_i] = d(\mu_i, \pi_i) \tag{50}$$

where π_i is the premium the insurer charges persons in risk group A_i and $d(x, y)$ is a demand function with $0 < d(x, y) < 1$. Without loss of generality, we will assume $0 < \mu_1 < \mu_2 < \ldots < \mu_n < 1$. The conditional independence means that insurance purchase is independent of the outcome — moral hazard is absent — although it is generally not independent of the risk of the outcome. Then from (48) above:

$$E[L] = \sum_{i=1}^{n} \mu_i p_i \tag{51}$$

$$E[Q] = \sum_{i=1}^{n} d(\mu_i, \pi_i) p_i \tag{52}$$

$$E[QL] = \sum_{i=1}^{n} d(\mu_i, \pi_i)\mu_i p_i. \tag{53}$$

Noting that the premium Π as a function of ω is itself a random variable, if the same premium π_i is charged to everyone in risk group A_i, we can by similar methods show that:

$$E[\Pi] = \sum_{i=1}^{n} d(\mu_i, \pi_i)\pi_i p_i. \tag{54}$$

Equations 51–54 provide formal justifications for Equations 1–4.

B. Loss Coverage Ratio

The loss coverage ratio for the case of equal demand elasticity is given in Equation 31 and can be expressed as follows:

$$C(\lambda) = \frac{1}{\pi_0^{\lambda}} \frac{\alpha_1\mu_1^{\lambda+1} + \alpha_2\mu_2^{\lambda+1}}{\alpha_1\mu_1 + \alpha_2\mu_2}, \quad \text{where} \quad \pi_0 = \frac{\alpha_1\mu_1^{\lambda+1} + \alpha_2\mu_2^{\lambda+1}}{\alpha_1\mu_1^{\lambda} + \alpha_2\mu_2^{\lambda}}; \tag{55}$$

$$= \left[w\mu_1^{\lambda-1} + (1-w)\mu_2^{\lambda-1}\right]^{\lambda} \left[w\mu_1^{\lambda} + (1-w)\mu_2^{\lambda}\right]^{1-\lambda} \quad \text{where} \quad w = \frac{\alpha_1\mu_1}{\alpha_1\mu_1 + \alpha_2\mu_2}; \tag{56}$$

$$= E_w\left[\mu^{\lambda-1}\right]^{\lambda} E_w\left[\mu^{\lambda}\right]^{1-\lambda}, \tag{57}$$

where E_w denotes expectation in this context and the random variable μ takes values μ_1 and μ_2 with probabilities w and $1-w$ respectively.

Result B.1. *For $\lambda > 0$,*

$$\lambda \lesseqqgtr 1 \Rightarrow C(\lambda) \gtreqqless 1. \tag{58}$$

Proof. **Case $\lambda = 1$:** It follows directly from Equation 57 that $C(1) = 1$.

Case $0 < \lambda < 1$: Holder's inequality states that, if $1 < p, q < \infty$ where $1/p + 1/q = 1$, for positive random variables X, Y with $E[X]^p, E[Y]^q < \infty$, $E\left[X^p\right]^{1/p} E\left[Y^q\right]^{1/q} \geq E[XY]$.

Setting $1/p = \lambda$, $1/q = 1-\lambda$, $X = \mu^{\lambda(\lambda-1)}$ and $Y = 1/X$, applying Holder's inequality on Equation 57 gives,

$$C(\lambda) = E_w\left[X^{1/\lambda}\right]^{\lambda} E_w\left[Y^{1/(1-\lambda)}\right]^{1-\lambda} \geq E_w[XY] = 1. \tag{59}$$

Case $\lambda > 1$: Lyapunov's inequality states that, for positive random variable μ and $0 < s < t$, $E[\mu^t]^{1/t} \geq E[\mu^s]^{1/s}$.

So Equation 57 gives:

$$C(\lambda) = \frac{E_w\left[\mu^{\lambda-1}\right]^{\lambda}}{E_w\left[\mu^{\lambda}\right]^{\lambda-1}} = \left[\frac{E_w\left[\mu^{\lambda-1}\right]^{1/(\lambda-1)}}{E_w\left[\mu^{\lambda}\right]^{1/\lambda}}\right]^{\lambda(\lambda-1)} \leq 1, \tag{60}$$

as $E_w\left[\mu^{\lambda-1}\right]^{1/(\lambda-1)} \leq E_w\left[\mu^{\lambda}\right]^{1/\lambda}$ for $\lambda > 1$ by Lyapunov's inequality.

□

Result B.2. *For* $0 < \lambda < 1$,

$$\max_{w} C(\lambda) = \frac{\beta - 1}{\beta^{\lambda(1-\lambda)} \left(\frac{\beta^{\lambda}-1}{\lambda}\right)^{\lambda} \left(\frac{\beta^{1-\lambda}-1}{1-\lambda}\right)^{1-\lambda}}, \quad \textit{where} \quad \beta = \frac{\mu_2}{\mu_1} > 1. \tag{61}$$

Proof. Proceeding from Equation 56, we have:

$$C(\lambda) = \frac{\left[w\beta^{1-\lambda} + (1-w)\right]^{\lambda} \left[w + (1-w)\beta^{\lambda}\right]^{1-\lambda}}{\beta^{\lambda(1-\lambda)}} \tag{62}$$

$$\Rightarrow \frac{\partial}{\partial w} \log C(\lambda) = \frac{\lambda(\beta^{1-\lambda} - 1)}{w\beta^{1-\lambda} + (1-w)} - \frac{(1-\lambda)(\beta^{\lambda} - 1)}{w + (1-w)\beta^{\lambda}} \tag{63}$$

$$\Rightarrow \frac{\partial^2}{\partial w^2} \log C(\lambda) = -\frac{\lambda(\beta^{1-\lambda} - 1)^2}{\left[w\beta^{1-\lambda} + (1-w)\right]^2} - \frac{(1-\lambda)^2(\beta^{\lambda} - 1)^2}{\left[w + (1-w)\beta^{\lambda}\right]^2} < 0. \tag{64}$$

$$\Rightarrow \frac{\partial}{\partial w} \log C(\lambda) = 0 \Rightarrow w = \frac{\lambda(\beta - 1) - (\beta^{\lambda} - 1)}{(\beta^{1-\lambda} - 1)(\beta^{1-\lambda} - 1)} \quad \text{gives the maximum.} \tag{65}$$

Inserting the value of w in Equation 62, gives the required result. □

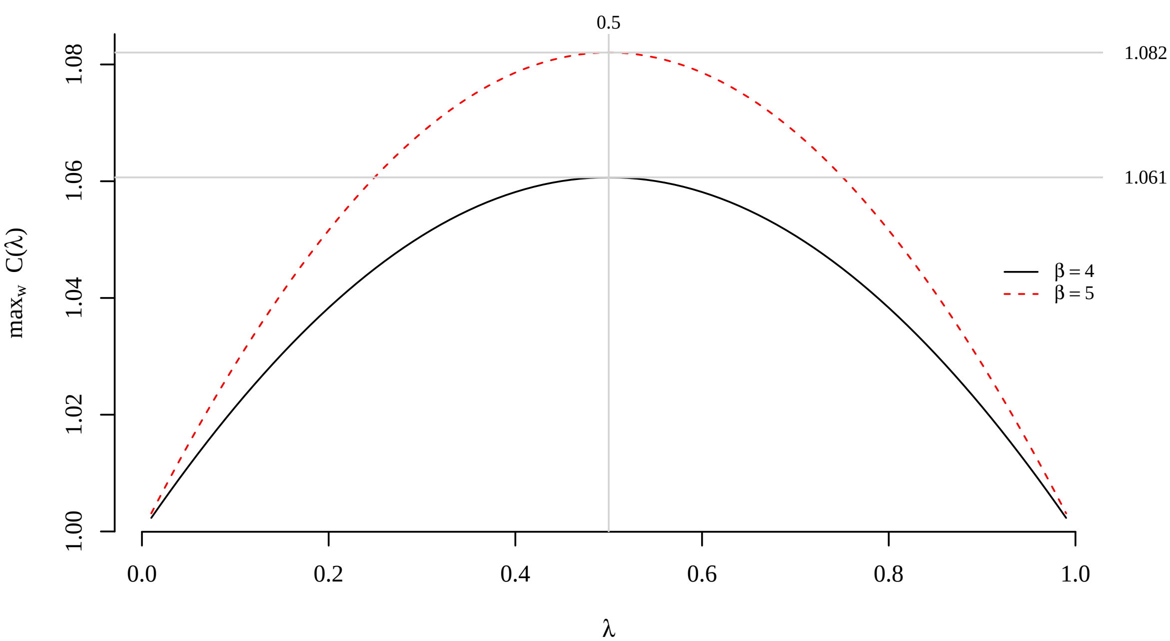

Figure 6: Maximum loss coverage ratio as a function of λ for specific values of β.

Figure 6 shows the plots of $\max_w C(\lambda)$ for $\beta = 4, 5$. This leads to the following result:

Result B.3. *For* $0 < \lambda < 1$,

$$\max_{w,\lambda} C = \frac{1}{2}\left(\sqrt[4]{\beta} + \frac{1}{\sqrt[4]{\beta}}\right). \tag{66}$$

Proof. Equation 61 can also be expressed as:

$$\max_{w} C(\lambda) = \frac{1}{2}\left(\sqrt[4]{\beta} + \frac{1}{\sqrt[4]{\beta}}\right) \frac{2\left(\sqrt[4]{\beta} - \frac{1}{\sqrt[4]{\beta}}\right)}{\left(\frac{(\sqrt{\beta})^{\lambda} - \frac{1}{(\sqrt{\beta})^{\lambda}}}{\lambda}\right)^{\lambda}\left(\frac{(\sqrt{\beta})^{1-\lambda} - \frac{1}{(\sqrt{\beta})^{1-\lambda}}}{1-\lambda}\right)^{1-\lambda}}, \tag{67}$$

$$= \frac{1}{2}\left(\sqrt[4]{\beta} + \frac{1}{\sqrt[4]{\beta}}\right) \frac{R(\frac{1}{2})}{R(\lambda)}, \tag{68}$$

$$\text{where} \quad R(\lambda) = \left(\frac{(\sqrt{\beta})^{\lambda} - \frac{1}{(\sqrt{\beta})^{\lambda}}}{\lambda}\right)^{\lambda}\left(\frac{(\sqrt{\beta})^{1-\lambda} - \frac{1}{(\sqrt{\beta})^{1-\lambda}}}{1-\lambda}\right)^{1-\lambda}. \tag{69}$$

The result follows from $R(\lambda) \geq R(\frac{1}{2})$, which in turn follows from the fact that $R(\lambda)$ is symmetric and convex over $0 < \lambda < 1$. As symmetry is obvious, we only need to prove convexity of $R(\lambda)$.

Note that,

$$\log R(\lambda) = g(\lambda) + g(1-\lambda), \quad \text{where} \quad g(\lambda) = \lambda \log\left(\frac{(\sqrt{\beta})^{\lambda} - \frac{1}{(\sqrt{\beta})^{\lambda}}}{\lambda}\right). \tag{70}$$

If $g(\lambda)$ is a convex function over (0,1), then $g''(\lambda) \geq 0$ and $g''(1-\lambda) \geq 0$, so $\log R(\lambda)$ is convex, which in turn implies $R(\lambda)$ is convex. So it suffices to show that:

$$g(x) = x \log\left(\frac{a^x - a^{-x}}{x}\right) \tag{71}$$

is convex over (0,1), where $a = \sqrt{\beta} > 1$. Now,

$$g'(x) = \log\left(\frac{a^x - a^{-x}}{x}\right) + \left(\frac{a^x + a^{-x}}{a^x - a^{-x}}\right) x \log a - 1. \tag{72}$$

$$g''(x) = \frac{(a^x + a^{-x})x \log a - (a^x - a^{-x})}{x(a^x - a^{-x})} + \frac{a^{2x} - a^{-2x} - 4x \log a}{(a^x - a^{-x})^2} \log a \geq 0, \tag{73}$$

as both $[(a^x + a^{-x})x \log a - (a^x - a^{-x})]$ and $[a^{2x} - a^{-2x} - 4x \log a]$ are increasing functions starting from 0 at $x = 0$. Hence proved. $\square$